

Reducing computational effort in the calculation of annual energy produced in wind farms

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Abstract

Metaheuristic methods are commonly used in the optimization of wind farms by means of turbine micro-siting. The typical pattern search used by these methods to explore the solution space makes it necessary to repeatedly evaluate the objective function (and hence the annual energy produced by the wind plant under optimization), a large number of times. For each case, before evaluating energy production, it is necessary to calculate the wind speed deficit at the position of each turbine due to the wake effect: a very time-consuming task.

This paper presents a set of algorithms and ideas to reduce the computational time spent on the calculation of the wakes, and therefore in assessing the annual energy production of a wind farm. The improvements proposed here can be applied to any wind turbine layout, but in the case of rhomboidal-type arrangements of turbines, their regularity leads to the achievement of much greater reductions in the computation effort. Furthermore, for this case, the computer time results almost independent of the number of turbines.

These improvements have been successfully applied to a set of cases, and show that the computation time for the calculation of the yearly energy production of a wind farm with 100 turbines can be reduced between 300 and more than 20 000 times, by using the most efficient of the strategies proposed.

Keywords: wind energy, wake effect, regular patterns

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1. Introduction

According to the Global Wind Energy Council [1], [2], a global capacity of 40.5GW of new wind power was installed in 2011, which represents investments of more than €50 billion (about \$68 billion), and an annual growth rate of about 6%. By the end of 2011, the global cumulative wind power capacity reached 237.7GW and the wind industry sector provided employment to about 650,000 people [2].

These figures show that nowadays, despite the current economic turbulence, wind energy remains an increasing area of research and development worldwide, since all the economic and social fundamentals which have driven the sector over the past decades are still in force: energy security at an affordable and stable price; local economic development and job creation; as well as the reduction of polluting, toxic and greenhouse gas (global warming) emissions.

To ensure the profitability of any wind farm project, especially in the case of the great offshore plants, it is necessary to harness the advantages of economies of scale. These benefits encourage developers to design wind farms with the largest possible number of turbines. This means that the optimal design tools for wind farms should be able to handle great quantities of wind generators. However optimizing a wind farm with a large number of turbines is a very time-consuming task. As shown in Section 2, the computation time spent on evaluating the yearly energy produced by a wind plant, which is the main factor in determining the rate of return of the investment, increases quadratically with the number of turbines.

Furthermore, most strategies seeking the optimal micro-siting for onshore or offshore wind plants are based on metaheuristic methods, which use evolutionary algorithms [3], [4] or some other kind of pseudo-random exploration of the solution space [5], [6], [7]. These techniques require assessment of a very large number of potential solutions until the optimal configuration is finally encountered. Since the calculation of the annual energy production of the wind farm is the most time-consuming task in the analysis of its profitability, it is clear that

reducing the time spent on this task would:

- Reduce the search time of the optimal wind farm layout.
- Enable a greater number of potential solutions in the same computation effort to be explored.
- Enable the complexity of the problem to be augmented, without increasing the computational cost.

One obvious way to speed up the calculation of the energy produced consists of considering a reduced number of directions and/or wind speeds, that is, by worsening the resolution of the considered wind rose or wind-speed field. Thus, a wind rose with a few sectors would be used, and the possible wind sources considered would be distributed in only 8 or 12 directions. This can lead to incorrect calculations, as in the case illustrated in Fig. 1.a, in which the erroneous result shows that there is no loss of production at all: this oversight is due to the exclusion of the wake effect for turbines aligned in directions not covered by the wind rose.

Figure 1 also shows a significant limitation of reviewed research studies [3], [4], [8], [9], which are derived from the discretization of the available land surface by means of a grid. The use of grids to discretize the available land allows the designer to reduce the computational cost of the calculation of the wake-induced loss of production. Since the grid restricts the potential turbine positions to the centre of the cells (or nodes) of the grid, it greatly reduces the space solution of the optimization problem, but at the expense of limiting the potential quality of the optimum solution. In order to illustrate the extraordinary increase in the size of the solution space (number of potential solutions) of the problem, N_{sol} , triggered by increasing the resolution of the discretization, the following example is proposed: assuming that the number of turbines to be installed at the wind farm, N_t , is known and that the area under study has been divided into a number of cells N_{cell} , then the number of possible solutions can be calculated as the number of combinations of N_{cell} taking N_t at a time without repetition,

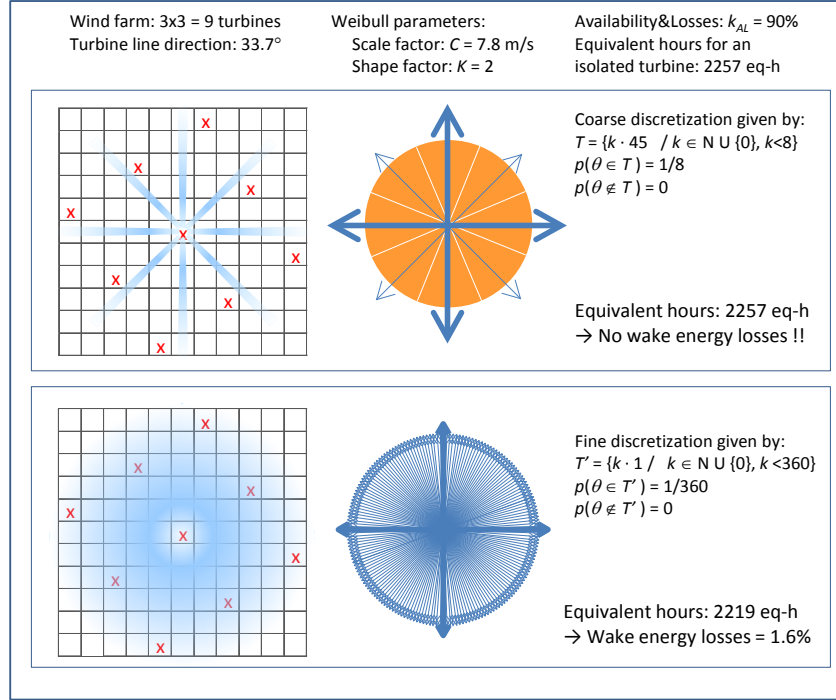


Figure 1: Comparison of the calculation of the annual production of energy of a wind plant considering a) coarse discretization (8 sectors) leading to incorrect results, and b) finely tuned discretization (360 sectors) of the wind rose.

and is given by the expression:

$$N_{sol} = \binom{N_{cell}}{N_t} = \frac{N_{cell}!}{N_t! (N_{cell} - N_t)!} \quad (1)$$

Assuming that the number of turbines to be installed at the wind farm depends on the available area, then by taking a ratio of wind turbines per cell, $r_{WT/C}$, the number of turbines can be expressed as:

$$N_t = r_{WT/C} N_{cell} \quad (2)$$

Table 1 shows the increase of the solution space when the size of the computational domain is varied (with constant values of the ratio of wind turbines per cell ($r_{WT/C} = 0.25$ and $r_{WT/C} = 0.5$)). From the results shown, it is possible to perceive the extraordinary increase in the complexity of the problem on increasing the resolution of the discretization.

Therefore, when optimizing the problem by means of population-based methods (as is usual with metaheuristic optimization methods) the size of this popu-

Table 1: Increase in solution space with increasing the number of cells in which the computational domain is discretized, considering two constant values of the ratio of wind turbines per cell.

Discretization (cells)	N_{sol} for $r_{WT/C} = 0.25$	N_{sol} for $r_{WT/C} = 0.5$
4×4	$1.82 \cdot 10^3$	$1.29 \cdot 10^4$
6×6	$9.41 \cdot 10^7$	$9.08 \cdot 10^9$
8×8	$4.89 \cdot 10^{14}$	$1.83 \cdot 10^{18}$
10×10	$2.43 \cdot 10^{23}$	$1.01 \cdot 10^{29}$
12×12	$1.13 \cdot 10^{34}$	$1.48 \cdot 10^{42}$

lation (the number of possible solutions to be evaluated in each iteration) has to be fitted to the size of the solution space so that a proper exploration of this solution space is assured. Under these circumstances, the guidelines and methods provided in this paper, whose aim is to reduce the computational effort when assessing the energy production of each possible wind farm configuration during the optimization process, constitute a key tool towards enabling the calculation of the optimal wind farm design since they assure adequate precision, related with the resolution of both wind rose and computational domain discretization.

After this introduction the content of the paper is organized as follows. Section 2 shows the general method for calculating the annual energy produced by a wind farm, by considering the energy losses due to the disturbance produced by an upstream turbine on those other turbines placed downstream (wake effect). This section also summarizes the underlying equations in the calculation of the perturbation exerted by a turbine in the wind speed of the flow through the turbine. In Section 3, a number of improvements to speed up the process of calculating the annual energy production are proposed. This section also shows the restrictions and limitations to be taken into account for the case that provides the biggest reduction in computation time. Section 4 provides a comparison of the results obtained, in terms of computation time, using the various improvements proposed. Finally, Section 5 summarizes the recommendations for the reduction of the computation effort needed for the calculation of the annual energy produced by a wind farm. The Appendix contains the pseudocode

corresponding to the improvements proposed.

2. Calculation of the annual energy produced by a wind farm

The general form for calculating the energy produced by a wind farm is obtained by adding together the energy, E_j , produced by each of the N_t turbines in the wind farm, for every wind direction and for every wind speed. In a year ($T = 8766$ h), the energy produced by the wind farm, E_{WF} , can be obtained from:

$$E_{WF} = k_{AL} T \sum_{j=1}^{N_t} E_j. \quad (3)$$

In this expression, the availability and losses factor, k_{AL} ($k_{AL} \leq 1$), allows the designer to take into account the loss of production due to unavailability of turbines and other losses including: those that occur in the electrical installation or due to delays and errors in the turbine orientation; errors in the power curve of the turbine itself; delays in turbine reconnection after a stop for exceeding the maximum speed (hysteresis). Obtaining the proper value of this parameter is not a trivial problem; however it can be estimated heuristically or, in the case of electrical losses, by analyzing the electrical infrastructure of the wind plant.

2.1. Discretizing the calculation of the annual energy produced by a turbine

In the case of an isolated turbine, j , the (theoretical) energy generated in a year, E_j , can be calculated from its power curve, $Pc(u)$, and the probability that the wind blows with speed u , $fr_{ws}(u)$ by using [10]:

$$E_j = k_{AL} T \int_{u_{cut-in}}^{u_{cut-out}} Pc(u) fr_{ws}(u) du \quad (4)$$

where u is the free-flow wind speed, T is the analyzed period of time (a year) and u_{cut-in} and $u_{cut-out}$ are the cut-in speed and cut-out speed of the studied wind turbine model, respectively.

In the case of a wind farm with N_t turbines arranged in a configuration, C , in which each turbine can suffer the perturbation of other upstream turbines, it

is necessary to include in the above expression the power deficit resulting from the perturbation. This deficit is likened to a deficit in the speed intensity, which causes the effective speed of the air flow that reaches the turbine $j-th$, v_j , to be smaller than the free-flow air speed, ($v_j < u$).

In order to complete the problem definition, it is necessary to determine the value of the actual wind speed, v_j , at the location of turbine $j-th$, due to the wake effect. The actual wind speed, v_j , depends on the free-flow wind speed, u , in the direction from which the wind blows, θ , as well as the relative position of the $j-th$ turbine with respect to the other turbines, C . This dependence can be expressed as:

$$v_j = u - Def(u, \theta, j, C). \quad (5)$$

where the function $Def(u, \theta, j, C)$ represents the wind speed deficit of the $j-th$ turbine due to the wake effect generated by upstream turbines. In this case, the annual energy produced by the wind farm is calculated as:

$$E_{WF} = k_{AL}T \sum_{j=1}^{Nt} \int_{u_{cut-in}}^{u_{cut-out}} \int_0^{2\pi} Pc(v_j) fr_{ws}(u) du fr_{dir}(\theta) d\theta \quad (6)$$

where $fr_{dir}(\theta)$ is the probability that the wind comes from the direction θ .

The perturbation function $Def(\cdot)$ is not analytically integrable, irrespective of whether it is possible to obtain an analytic expression for the functions $Pc(\cdot)$, $fr_{ws}(\cdot)$ or $fr_{dir}(\cdot)$, and hence (6) has to be discretized:

$$E_{WF} = k_{AL}T \sum_{j=1}^{Nt} \sum_{s=1}^{Ns} \sum_{v=1}^{Nv} Pc(v_j) fr_{ws}(u_v) fr_{dir}(\theta_s) \Delta u \Delta \theta \quad (7)$$

In this expression:

- N_v is the number of equal intervals in which the considered wind speed interval $[v_{cut-in}, v_{cut-out})$ is divided.
- N_s is the number of sectors taken into account for the evaluation of the incoming wind directions (the number of sectors of the considered wind rose).

- $\Delta u = (v_{cut-out} - v_{cut-in})/N_v$ is the resolution used for wind speeds.
- $\Delta\theta = 2\pi/N_s$ is the resolution used for the incoming wind directions, that is, the angular size of each sector of the considered wind rose.
- $fr_{ws}(u_v)T$ is the number of hours per year that the wind blows at a speed u , between u_v and $u_v + \Delta u$, i.e. $u \in [u_v, u_v + \Delta u)$.
- $fr_{dir}(\theta_s)T$ is the number of hours per year that the wind comes from sector s of the considered wind rose.

Logically $fr_{dir}(\theta_s)fr_{ws}(u_v)T$ is the number of hours per year that the wind blows at a speed between u_v and $u_v + \Delta u$, coming from sector s of the considered wind rose.

2.2. Calculation of the power production deficit

Figure 2 shows the evolution of the wake behind a turbine that affects a downstream turbine. The equations for the calculation of the speed deficit at the downstream turbine are taken from [11], [12], but starting with an initial expansion of the wake section provided by $\beta\pi r_0^2$, and are summarized in the following expressions:

$$D_w(x) = \sqrt{\beta}D_0 \left(1 + 2\alpha_{noj} \frac{x}{D_0} \right) \quad (8)$$

$$\beta = \frac{1}{2} + \frac{1}{2\sqrt{1-C_t}} \quad (9)$$

$$\Delta v = u - v = \frac{1}{2} - \frac{1}{2} \sqrt{1 - C_t \left(\frac{D_0}{D_w} \right)^2} = \quad (10)$$

$$\frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{C_t}{\beta} \left(\frac{1}{\left(1 + 2\alpha_{noj} \frac{x}{D_0} \right)^2} \right)}.$$

In the previous expressions, $D_w(x)$ is the diameter of the wake at a distance x behind the upstream turbine (distance T_1 and T_{2A}), D_0 (r_0) is the turbine rotor diameter (radius), α_{noj} is an non-dimensional parameter experimentally

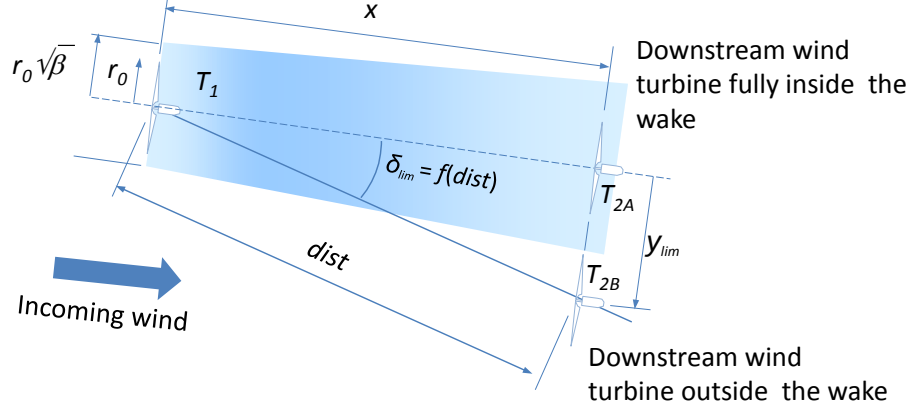


Figure 2: Expansion of the wake due to an upstream turbine. Angle δ_{lim} between the wind direction and the line between turbines beyond which there is no perturbation at all.

obtained ($\alpha_{noj} = 0.05$ according to [12], [13]), β is the entrainment constant ($\beta > 1$) relative to the initial expansion of the wake which is obtained from the thrust coefficient C_t , and Δv is the speed deficit at the downstream turbine. When the turbines are not aligned with the wind direction (T_1 and T_{2B} in Fig. 2), the variable x is the projection of the distance between turbines, $dist$, over the incoming wind direction.

The thrust coefficient, C_t , can be obtained from the following set of instructions [10]:

$$\begin{aligned} C_p &= \frac{Pc(v)}{\frac{1}{2}\rho v^3 S} \\ C_p &= 4a(1-a)^2 \\ C_t &= \frac{C_p}{1-a} \end{aligned} \quad (11)$$

where C_p is the power coefficient, Pc is the power obtained from the turbine power-speed curve, ρ is the air density, v is the wind speed, S is rotor area and a is the inflow coefficient.

Symmetry In the case of partial wake, the calculation of the overlapping area can be found in [4]. By naming the speed deficit caused by the upstream turbine j on downstream turbine i as $\Delta v_i^j = u - v_i^j$, then the wind speed deficit in the downstream turbine i due to the combination of the wakes can be expressed as the root square of the quadratic sum of each individual deficit,

separately calculated [14].

$$u - v_i = \sqrt{\sum_{i \neq j} (u - v_i^j)^2} \quad (12)$$

2.3. General method for calculating the yearly energy production of a wind farm

Appendix A.1 shows the basic pseudocode corresponding to the routine to be performed in order to calculate the annual energy production of each turbine. This code is applicable to any turbine layout, and not only for those where the positions of the turbines is restricted to certain grid positions. As can be seen, the number of operations to be performed in order to calculate the total yearly energy production of the wind farm is:

$$n_{op} = N_t(N_t - 1) \cdot N_s \cdot N_v. \quad (13)$$

Accordingly, the computer time, proportional to the number of operations, grows with the square of the number of turbines.

For example, considering a wind farm with $N_t = 50$ turbines, an angular discretization for the wind directions $N_s = 360$ (corresponding to $\Delta\theta = 1^\circ$), and a wind speed discretization, $N_v = 21$ (corresponding to a turbine practical wind speed interval $u_{cut-out} - u_{cut-in} = 21 \text{ m/s}$ and $\Delta u = 1 \text{ m/s}$), the number of operations is $1.85 \cdot 10^6$.

2.4. **Action 0.** Easily programmable first actions

In this Section, a series of simple improvements that prevent useless calculations are introduced. These basic improvements are discussed below in the three following subsections.

2.4.1. Disregarding the wake effect between two turbines placed very far apart

As the distance between two turbines increases, x in Fig.2, the wind speed deficit due to wake effect decreases. This enables a threshold on the power deficit, ΔP_{th} , to be established which corresponds to a threshold of distance, tr_d , from which the wind speed deficit due to the wake effect can be disregarded.

That is, from the threshold of distance upwards ($x > tr_d$), the power deficit $\Delta P(v(x))$ is less than a certain value, ΔP_{th} . Formally,

$$\text{given } \Delta P_{th}, \exists tr_d : \Delta P(v(x)) \equiv P(u) - P(v(x)) < \Delta P_{th} \quad \forall x > tr_d. \quad (14)$$

Thus, the perturbation of an upwind turbine over another downstream turbine placed at a distance greater than threshold, tr_d , can be neglected. Furthermore, the threshold distance, tr_d , can be calculated a priori, since it only depends on the incoming free-flow wind speed.

2.4.2. Disregarding wind directions when the deviation angle exceeds δ_{lim}

According to Fig. 2, δ is the deviation angle between wind direction and the line alignment between two turbines. Deviations higher than δ_{lim} produce no power deficit. The angle δ_{lim} can be obtained from the following expressions:

$$\begin{aligned} y_{lim} &= r_0 (1 + \sqrt{\beta}) + \alpha_{noj} \sqrt{\beta} x \\ dist^2 &= y_{lim}^2 + x^2 \\ \delta_{lim} &= \cos^{-1} \frac{x}{dist} \end{aligned} \quad (15)$$

As can be observed, x can be obtained from $dist$ and β , and therefore the angle δ_{lim} depends on the distance, $dist$, and the incoming wind velocity through β .

2.4.3. Disregarding very high wind speeds

Any downstream turbine could be affected by the upstream turbine wake, but its wind speed deficit (at the downstream turbine) should not exceed 20%-25% of the free-flow speed, in a well-designed wind farm. In the case where the speed deficit exceeded that limit, the optimization algorithm itself would without doubt, rule out that solution.

In addition, the power curves of current turbines show that for each turbine model, there is a speed, v_{Pr} , from which the supplied power is constant and equal to the turbine rated power ($v \geq u_{Pr} \Rightarrow P(v) = P_r$). In this case, it can be assumed that for any free-flow wind speed sufficiently higher than u_{Pr} , the effective speed of the turbine downstream, v , remains higher than that of

the affected v_{Pr} (despite the speed deficit due the wake) and, consequently, the power generated is the maximum: the rated value.

1.25 This way, if the algorithm designer consider that a speed deficit higher than 25% is not acceptable in the optimum park layout, a limit value of $u_{LH} = 1.25u_{Pr}$, can be established. Accordingly, it is useless to calculate power deficits at speeds above u_{LH} since the power production of the downstream turbine remains at its rated value.

1.25 The value of 1.25 is a trade-off between accuracy and time reduction: selecting a lower value, the energy calculation will loose accuracy for configurations in which the turbines are not so close; a higher value will cause a higher computational time.

3. Rules for the reduction of computational time

In addition to the simple improvements mentioned above, there are other, more complex, improvements that can reduce the computation time/effort required to calculate the yearly energy production of a wind plant, while taking into account the effect of the wakes. These actions are aimed at:

- **Prevention of repeating previously made calculations.** These refer to the fact that when evaluating the energy produced by the wind there are a number of calculations that need to be made but that are repeated several times with identical or very similar parameters. This Rule encompasses new *Actions* 1 and 2.
- **Suppression of unnecessary calculations where no perturbation is expected.** The purpose is to include decisions that would normally be taken by an expert to suppress calculations in situations in which no disturbance exists. This Rule encompasses new *Actions* 3 and 4.
- **Harnessing of regular layout of the wind farm to identify recurring situations.** This Rule corresponds to new *Action* 5.

3.1. **Action 1.** Storing turbine positions

As indicated in the pseudocode of the Appendix A.1 and considering a discretization of N_s sectors for wind directions and N_v for wind speeds, the position of each turbine is calculated $N_t \cdot N_s \cdot N_v$ times. Accordingly, the storage of the position of each of the turbines in an array of N_t elements is expected to reduce the number of calculations.

3.2. **Action 2.** Storing speed deficits

The yearly energy produced by the wind farm can be calculated by adding together the power produced by each of its turbines, while taking into account the disturbing effect (speed deficit) produced by other turbines that are upstream in the direction of the incoming wind. This effect depends on the following factors (see eqs. (8)-(11)):

- The normalized distance between the turbines, defined as $d_{pu} = dist/D_0 = dist/(2r_0)$ in Fig.2.
- The deviation angle, δ , between the line connecting both turbines and the direction of the incoming wind.
- The parameter β , which is a function of the thrust coefficient, C_t . The coefficient C_t , in turn, depends on the free-flow wind speed and on the turbine power curve.

The strategy in this case would establish a discretization for these input values: normalized distance, deviation angle and free flow speed; and complete a three-dimensional look-up table as Symmetry each input 3-tuple appears in the search algorithm. This way, the individual speed deficits Δu_i^j that are necessary for obtaining the overall speed deficit over the turbine i (see expression (12)) are calculated only once for each discretized input 3-tuple (d_{pu}, δ, β) .

Table 2 shows the discretization values used in the set of test cases considered in this paper.

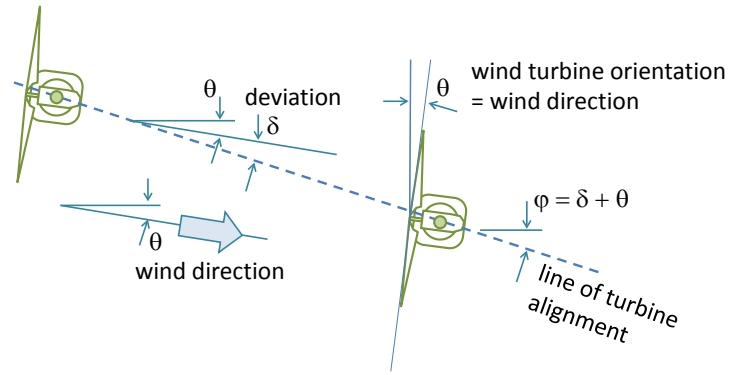


Figure 3: Angles to be taken into account for the calculation of the produced energy

Table 2: Look-up table with stored speed deficit values. Range for the input parameters.

variable	Min	Max	Resolution
Distance range	$4 D_0$	$30 D_0$	$0.1 D_0$
Angle δ	0°	20°	1°
u	u_{cut-in}	$u_{cut-out}$	0.1 m/s

3.3. **Action 3.** *Deriving information from previous calculations*

Symmetry In the same way that in *Action 2*, the aim is to avoid calculations of individual deficits Δv_i^j that have been previously performed: since some of the addends of expression (12) have already been calculated and stored, the required time to compute the overall speed deficit is reduced. Unlike *Action 2*, this *Action* establishes the conditions in which two different deviations δ give rise to the same individual deficit.

As can be seen in Fig. 3, δ is the angle of deviation between the line alignment of two turbines and the incoming wind direction. Therefore, if there is no disturbance between these two turbines for this angle δ , there will be no disturbance for an angle $\delta_1 = \delta + |\Delta\theta|$ (or any positive increment), nor for $\delta_2 = -\delta_1 = -\delta - |\Delta\theta|$. **Formally**

$$\Delta v_i^j(d_{pu}, \delta, \beta) = 0 \Rightarrow \Delta v_i^j(d_{pu}, \delta', \beta) = 0, \quad \text{with } 0 < \delta < \delta' \leq \pi \quad (16)$$

Furthermore, by symmetry, the deficit for a direction deviated at an angle δ is the same as for a direction deviated at an angle $-\delta$

$$\Delta v_i^j(d_{pu}, |\delta|, \beta) = \Delta v_i^j(d_{pu}, -|\delta|, \beta) \quad (17)$$

Considering this action means changing the order of the nested loops in the pseudocode of the subappendix Appendix A.1, and using arrays to store intermediate values, specifically the quadratic deficit of each of the shadowed turbines (`sum_deficit2` in the aforementioned pseudocode). Appendix A.2 shows the simplified pseudocode for Action 3 which is supported in Figures 2 and 3.

3.4. **Action 4.** *Initial exploration at u_{crit}*

This subsection shows that there is a speed, u_{crit} , which is the most demanding in terms of expansion of the wake and magnitude of the disturbance (speed and power deficits). In the case when for a given free-flow wind speed, u_{crit} , and for a given orientation of the wind, a turbine T_p is outside the wake

produced by another turbine T_q , then it also stays outside their wake for any other magnitude of the free flow wind speed. Formally:

$$\begin{aligned}
I_u &\equiv [u_{cut-in}, u_{cut-out}) \\
\exists u_{crit} \in I_u & : D_w(x, u) \leq D_w(x, u_{crit}) \quad \forall u \in I_u \\
& : \Delta v(x, u) \leq \Delta v(x, u_{crit}) \quad \forall u \in I_u
\end{aligned} \tag{18}$$

To prove the existence of this speed, u_{crit} , we will start from expressions 8, 9 and 11 which provide the wake diameter, $D_w(x)$, and the speed deficit, Δv . That latter expression can also be expressed as:

$$\Delta v = \frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{C_t}{\beta} \left(\frac{1}{\left(1 + 2\alpha_{noj} \frac{x}{D_0}\right)^2} \right)}. \tag{19}$$

Given that $0 < C_t < 1$, it is noted from (9) that β is defined and it increases with C_t . In addition, it can be verified that the ratio C_t/β increases with C_t .

$$\frac{d\beta}{dC_t} > 0 \tag{20}$$

$$\beta > 1 \Rightarrow \frac{d\frac{C_t}{\beta}}{dC_t} > 0. \tag{21}$$

Equation (8) implies that the wake diameter D_w grows as β increases. Therefore, and according to (20), the wake diameter D_w also increases with the thrust coefficient, C_t .

Furthermore, (9) determines that the speed deficit, Δu , grows as C_t/β increases. Therefore, and according to eq. (21), the speed deficit, Δv , also increases with the thrust coefficient, C_t .

Assuming $u_{Ct\ max}$ is the speed at which the thrust coefficient, C_t , reaches its maximum value, it can be stated that:

- If a downstream turbine lies outside the wake defined by $D_w(x)$ for a wind speed equal to $u_{Ct\ max}$, then the turbine stays outside the wake for any speed.

- If a downstream turbine lies within the wake defined by $D_w(x)$ for a free flow wind speed equal to $u_{Ct\ max}$, but the wind speed deficit is less than a certain threshold ($\Delta v(u_{Ct\ max}) < \Delta u_{th}$), then the wind speed deficit remains less than the threshold for any other speed ($\Delta v < \Delta u_{th}$).

This wind speed is, logically, the one previously called u_{crit} . Accordingly, it is desirable to evaluate the existence of disturbance (the wind speed deficit due to the wake) only at the wind speed $u_{Ct\ max}$. If, under these conditions ($u = u_{crit} = u_{Ct\ max}$), the wind speed deficit is less than a certain threshold, Δv_{th} , then it is unnecessary to evaluate the remainder of the wind speeds, and results in a reduction of computational effort. This wind speed, $u_{crit} = u_{Ct\ max}$, depends on the particular turbine power curve, but is typically close to 8 m/s.

3.5. **Action 5.** *Harnessing wind farms with rhomboidal layouts*

In the common case where the turbines of the wind farm are located at the nodes of a rhomboidal grid, it is possible to group the turbines according to a set of alignments following several “line directions”. Figure 4 illustrates this idea for a wind plant with 5 rows and 7 columns. An alignment is a straight line in which there are two or more turbines aligned. The maximum number of equal segments in an alignment, mns , is equal to the number of aligned turbines minus one.

One possible line direction comes from advancing two positions in a row and one position within a column ($\Delta c = 2, \Delta r = 1$). Aligned in this direction, there are 2 lines with 3 segments, depicted in solid (blue) lines; five lines with 2 segments, in dashed (red) lines; and 4 lines with 1 segment, in dotted (green) lines. However this is only one of the possible line directions. The remaining line directions are: ($\Delta c = 0, \Delta r = 1$); ($\Delta c = 1, \Delta r = 0$); ($\Delta c = 1, \Delta r = 1$); ($\Delta c = 1, \Delta r = -1$); ($\Delta c = 1, \Delta r = 2$); ($\Delta c = 1, \Delta r = -2$) ... and so on.

In general terms, the possible line directions are given by the set of 2-tuples:

$$\begin{aligned} & \{(\Delta c = 0, \Delta r = 1), (\Delta c = 1, \Delta r = 0)\} \cup \\ & \cup (\Delta c = p, \Delta r = q) \in \mathbb{N} \times \mathbb{Z} - \{0\} \ / \ p \text{ and } |q| \text{ have no common factors } \neq 1 \end{aligned} \quad (22)$$

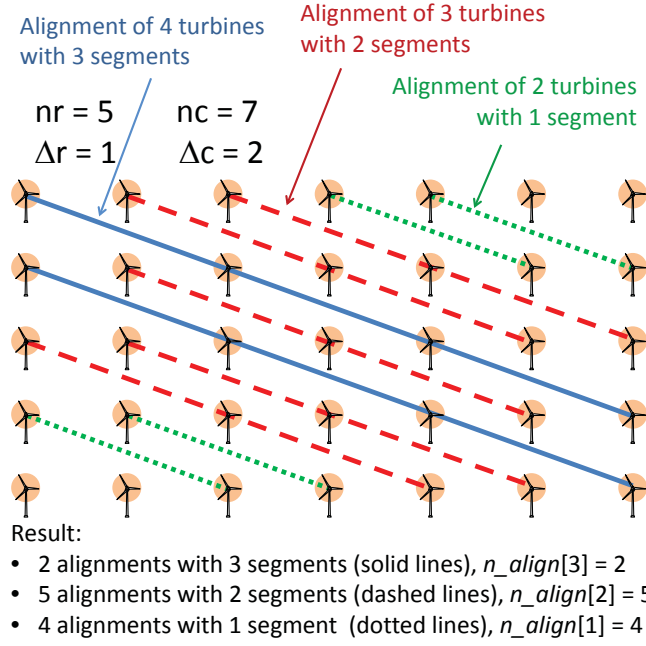


Figure 4: Alignments and segments in a wind farm. Maximum number of segments (mns) for a specific line direction, and number of alignments with 1 segment, with 2 segments,... and with mns segments.

The following subsections will show the algorithms for:

- The systematic way to obtain the number of alignments with 1 segment, with 2 segments, and so on, for each line direction.
- The calculation of the energy produced by analyzing, separately, each of these line directions.

3.5.1. Analyzing a specific line direction

The pseudocode of Appendix A.3 describes operations which enable the maximum number of equal segments, mns , that may exist along this alignment to be obtained, for a given line direction. In addition, the algorithm should calculate how many lines contain mns segments, how many lines contain $mns - 1$ segments, and so on, until finally the number of lines which contain only 1 segment is calculated. As an example, Fig. 4 shows the result for the case of an orientation given by $(\Delta c = 2, \Delta r = 1)$.

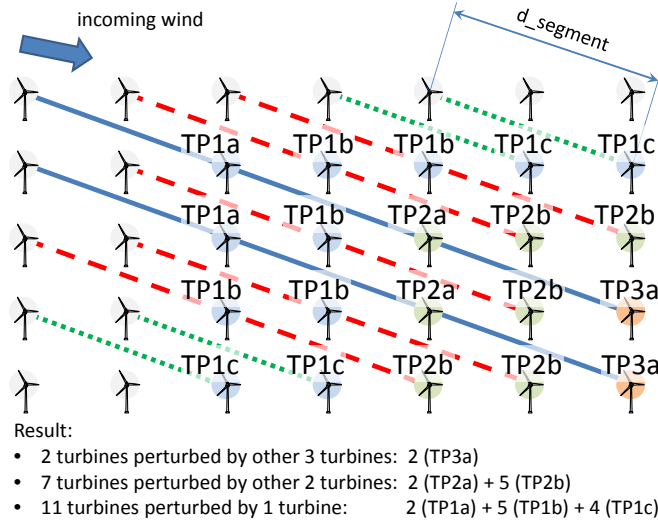


Figure 5: Turbines that suffer perturbation from 1 upstream turbine (TP1), from 2 upstream turbines (TP2) and from 3 upstream turbines (TP3). Suffixes *a*, *b* and *c* has been added to refer to lines with 3, 2 or 1 segment, respectively.

3.5.2. Calculating the annual energy production of a wind farm from its line directions

Appendix A.4 shows the pseudocode to calculate the annual energy production of a wind farm with a regular rhomboidal pattern of turbines (see Fig. 4 for a better understanding).

Assuming that for a line direction there are a maximum of mns segments, the procedure is based on determining, for that specific line direction:

- The number of turbines that are perturbed by only one turbine (the TP1 turbines), for 2 turbines (the TP2 turbines), ... and finally, by mns turbines.
- The yearly energy produced by each of these kinds of turbines.

Of course, certain aforementioned improvements can be added to this pseudocode.

Further to the obvious limitation that the method can only be used with rhomboidal turbine layouts, Fig. 6 presents another limitation. Figure 6.a shows the discrepancy between the energy calculated by means of general method,

E_{gen} , and by the proposed *Action 5*, E_{rhomb} , calculated as

$$\epsilon = \frac{E_{rhomb} - E_{gen}}{E_{rhomb}} = 1 - \frac{E_{gen}}{E_{rhomb}} \quad (23)$$

for a wind farm with $6 \times 8 = 48$ turbines. The various curves in the figure correspond to several values of the tilting angle, that is, the complementary of the angle between two adjacent sides of the rhomboid. As can be seen, the shorter the distance between turbines, and the higher the tilting angle then the greater the error; that is, the further the layout lies from the rectangular pattern. However, this error always remains lower than 1%.

This error is due to the fact that when excessively bending the rectangular shape to take the form of rhomboid, the wakes of an alignment interfere with turbines not only belonging to its own alignment, but to another parallel adjacent one (see Fig 6.b). This situation is not covered by the pseudocode in Appendix A.4.

However, in a practical situation, this limitation is not an important concern in the case of search algorithms for the optimal solution, since a wind farm with an excessively high tilting angle and a reduced distance between turbines would have foreseeably high wake-effect losses; accordingly, it would not be an optimum solution to the problem. In this case, the annual energy production may be obtained by the general method applicable to all types of layouts.

4. Comparison between the performances of various actions

Both the general method of calculation of the yearly energy production of a wind farm and the set of improvements provided by the several actions have been programmed in managed C++ language and executed on a notebook with a microprocessor Intel Pentium 2.13 GHz. The computation time using the generic algorithm for the calculation of the annual energy production of a wind farm with 100 turbines is about 36 s, and this computation time is proportional to the square of the number of turbines.

It is worth noting that the basic generic calculation algorithm, already includes the set of improvements encompassed in the so-called *Action 0*. More

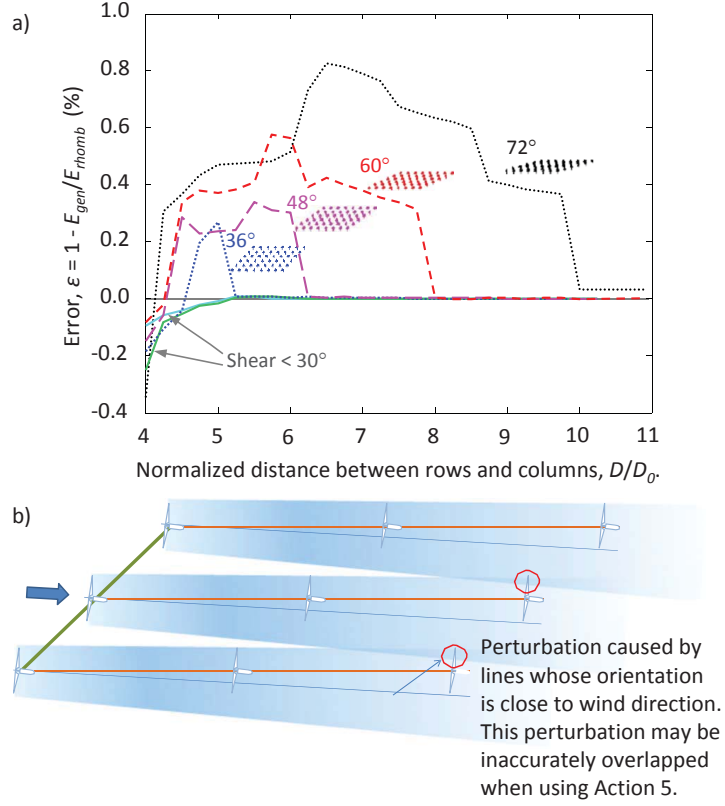


Figure 6: Wind farm with $N_t = 6 \times 8$ turbines. a) Discrepancy between the energy calculated by means of general method, E_{gen} , and the Action 5, E_{rhomb} , measured in % as $\epsilon = (E_{rhomb} - E_{gen})/E_{rhomb}$, b) illustration of the source of error when using Action 5 with lain rhomboids.

interesting than the computation time in absolute terms (which depends on the characteristics of the microprocessor and the programming language), is the percentage of time saved by using the improvements provided by the various proposed actions. Figure 7 shows the reduction of the computation time for the various improvements suggested. In this figure the time reduction ratio is defined as:

$$Time\ ratio_n = \frac{Time(Action\ n)}{Time\ without\ improvements} \quad (24)$$

Table 3 indicates the specific actions corresponding to each curve of the

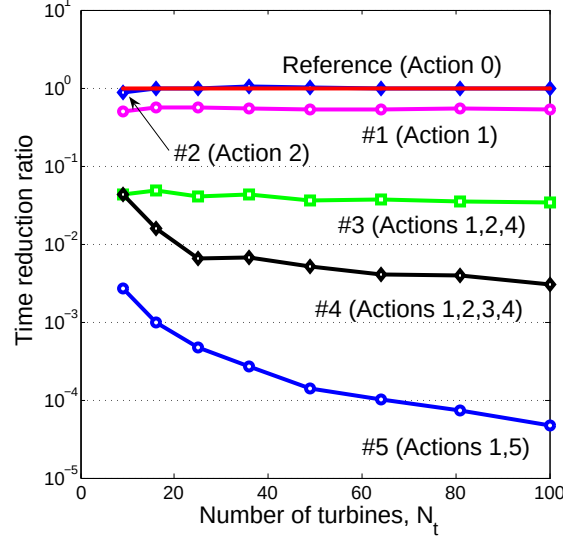


Figure 7: Time reduction ratio ($\text{Time_Action}_n / \text{Time_basic_algorithm}$) as a function of the number of turbines. Reduction of the computation time when using the various improvements, regarding the algorithm described in Section 2.

Table 3: Computer time and reduction time ratio for wind farms with $N_t = 49$ and $N_t = 100$ turbines. Actions taken into account for each of the plots in Fig. 7 and corresponding time reduction ratios.

Plot	Action					Time (ms)		Time ratio	
	1	2	3	4	5	$N_t=49$	$N_t=100$	$N_t=49$	$N_t=100$
Ref	No	No	No	No	No	9031	36 453	1.00	1.00
1	Yes	No	No	No	No	4828	19 250	0.53	0.53
2	No	Yes	No	No	No	9062	36 140	1.00	0.98
3	Yes	Yes	No	Yes	No	328	1234	$3.63 \cdot 10^{-2}$	$3.39 \cdot 10^{-2}$
4	Yes	Yes	Yes	Yes	No	46	109	$5.09 \cdot 10^{-3}$	$2.99 \cdot 10^{-3}$
5	Yes	No	No	No	Yes	1.25	1.71	$1.38 \cdot 10^{-4}$	$4.69 \cdot 10^{-5}$

graph. It also shows the *Time reduction ratio* achieved for cases of wind farms with 49 and 100 turbines.

As can be seen in Figure 7 and Table 3, the main improvement comes from the incorporation of *Action 5 (Harnessing rhomboidal layouts)*, although this improvement is only possible for regular rhomboidal patterns. Furthermore, the computer time results almost independent of the number of turbines. In this case, the calculation of the energy produced by the wind farm was reduced up to 1-2 ms, regardless of the number of turbines in the wind farm, for wind plants whose size ranged between 9 and 100 turbines.

Figure 7 also shows that the Time reduction ratio corresponding to Actions 1, 2 and 4 (plots #1 to #3) are independent of the number of turbines. This

means that the computer time corresponding to these Actions grows with the squared of the number of turbines, as does the Reference case. The hyperbolic shape of the Time reduction ratios corresponding to Actions 3 and 5 (plots #4 and #5) means that the computation time corresponding to these Actions grows less than quadratically with the number of turbines.

For general (not rhomboidal) arrangements, *Action 3 (Deriving information from previous calculations)* also represents a substantial reduction of computation time. Another feature of this *Action* is the fact that the increase in the computation time is not as much as a quadratic ratio, as it happens with the general procedure, and the *Action 1* and *Action 4*. These Actions 1 and 4 also give rise to an interesting computational improvement. However *Action 2 (Speed deficits stored in a table)* hardly reduces the computation time at all and involves significant changes to the code, which renders this action not be as profitable as would be expected.

Regarding additional RAM memory requirements for a wind farm with 100 turbines, *Action 3* requires 6 MB, while a mere 1.1 MB suffices for the remainder of the Actions.

5. Conclusions

The problem of finding the optimum layout for the turbines on a wind farm requires the use of a routine of evaluation of the annual energy production as time- and memory-efficient as possible in order to accomplish a complete and efficient exploration of the solution space that guarantees the quality of the optimum solution. This difficulty is usually dealt with by discretizing the problem, mainly the number of sectors of the wind rose and wind speeds, together with the implementation of some kind of pattern or grid to limit the possible positions of the turbines on the wind farm parcel. This approach to the problem, while allowing limitations to be placed on the computational effort (time) for the optimal layout configuration, restricts the quality of the optimum solution obtained, leading to misleading or inaccurate results in extreme cases.

This paper introduces a set of measures (Actions), most of which can greatly reduce the computation time required to calculate the annual energy produced by a wind farm while taking into account the effect of the wakes. In the case of a general arrangement of the turbines, the computational time reduction ratio with the number of turbines is stabilized around a factor of $3 \cdot 10^{-3}$ (more than 330 times faster), relative to the time spent by the reference algorithm, which already includes some simple improvements in computing the yearly production of energy. In the case of wind power plants with turbines in a rhomboidal-shaped layout, the computation time required (not the reduction of the calculation time but the total time) results almost independent of the number of turbines of the wind farm. In this case the cut in the time reduction ratio is much higher: a factor of $4.7 \cdot 10^{-5}$ (more than 21 000 times faster), for a wind farm with $N_t = 100$ turbines.

Regarding additional RAM memory requirements, including these improvements requires a storage memory capacity slightly higher, but overall, this increment is not significant, typically lesser than 1.1 MB and always lesser than 6 MB, for a wind farm with 100 turbines.

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Appendix A. Pseudocode

Appendix A.1. Basic pseudocode for the calculation of the energy produced by a wind farm

```

Function Energy_Produced
energy  $\leftarrow$  0
    % For each of the 360 wind directions
For all  $\theta$  from 0 to 359 Do
    % Frequency at this direction
Compute  $fr_{\theta} \leftarrow$  Direction_2_Frequency( $\theta$ )
    % For each of the usable wind speeds
For all ws from  $v_{cut\_in}$  to  $v_{cut\_out}$  Do
    % Frequency at this wind speed
Compute  $fr_{ws} \leftarrow$  WindSpeed_2_Frequency(ws)
    hours  $\leftarrow fr_{\theta} \cdot fr_{ws} \cdot 8766$  hours/year
    deficit  $\leftarrow$  0
    % For each of the nt "shadowed" turbines
For all i from 0 to nt-1 Do
    % Wind speed deficit at turbine i
    sum_deficit2  $\leftarrow$  0
    % For each of the (nt-1) possible "shadower" turbines
For all j from 0 to nt-1,  $i \neq j$  Do
    Compute  $\varphi \leftarrow$  angle of segment linking turbines Ti and Tj
    % deviation ( $\delta$ )  $\leftarrow$  wind_direction ( $\theta$ ) - line_orientation ( $\varphi$ )
     $\delta \leftarrow \theta - \varphi$  % see Fig. 3
    Compute dist  $\leftarrow$  length of segment linking turbines Ti and Tj
    Compute deficit  $\leftarrow$  Wind_Speed_Deficit( $\delta, ws, dist$ )
    Compute affected_area  $\leftarrow$  Affected_Area( $\delta, ws, dist$ )
    Increment sum_deficit2 By (deficit  $\cdot$  affected_area)2
End For
    % Obtain the effective wind speed seen by turbine i
 $v_{ef} \leftarrow ws - \sqrt{sum\_deficit2}$ 
    % Generated power at the effective speed
Compute P  $\leftarrow$  WindSpeed_2_Power( $v_{ef}$ )
    energy  $\leftarrow$  energy + P  $\cdot$  hours
End For
End For
Return energy
End Function

```

Appendix A.2. Pseudocode for the implementation of Action 3

```

Function Energy_Produced
    park_energy  $\leftarrow$  0
    % For each of the nt "shadowed" turbines
    For all i from 0 to nt-1 Do
        % Wind speed deficit at turbine i
        sum_deficit2  $\leftarrow$  0
        Clear Matrix deficit ( $Nt \times Ns \times Nv$ )
        % For each of the (nt-1) possible "shadower" turbines
        For all j from 0 to nt-1,  $i \neq j$  Do
            Compute  $\varphi \leftarrow$  angle of segment linking turbines Ti and Tj
            Compute dist  $\leftarrow$  length of segment linking turbines Ti and Tj
            % For each of the wind direction around  $\varphi$ 
            Compute  $\delta_{lim}$  From expressions (15)
            For all  $\delta$  from 0 to  $\delta_{lim}$  Do
                 $\theta_1 \leftarrow \varphi + \delta$ 
                 $\theta_2 \leftarrow \varphi - \delta$ 
                For all ws from  $v_{cut-in}$  to  $v_{cut-out}$  Do
                    Compute deficit  $\leftarrow$  Wind_Speed_Deficit( $\delta, ws, dist$ )
                    Compute affected_area  $\leftarrow$  Affected_Area( $\delta, ws, dist$ )
                    Compute deficit $_{\theta_1, j, ws}$ 
                    Compute deficit $_{\theta_2, j, ws}$ 
                End For
            End For
            Compute energy_of_turbine $_i$  from  $fr_{\theta}$ ,  $fr_{ws}$  and deficit
            % 3 nested loops:  $\theta$ , ws and j
            Increment park_energy By energy_of_turbine $_i$ 
        End For
    End For
    Return park_energy
End Function

```

Appendix A.3. Pseudocode for analyzing line directions in a rhomboidal pattern

```

Function Analyze_Alignments_In_Direction(nr, nc,  $\Delta r$ ,  $\Delta c$ )
    % Output parameters:
    % Array  $n\_align$ :  $n\_align_i$  is the number of alignments with i segments
    % Maximum number of segments mns

Clear  $n\_align$ 
If  $\Delta c = 0$  Do      % Line in the direction of columns
    mns  $\leftarrow$  nr-1
     $n\_align_{mns} \leftarrow$  nc      % nc alignments with mns segments
    Return mns and  $n\_align$ 
Else If  $\Delta r = 0$  Do
    mns  $\leftarrow$  nc-1
     $n\_align_{mns} \leftarrow$  nr
    Return mns and  $n\_align$ 
End If

ns_c  $\leftarrow$  (nc-1)/ $\Delta c$       % 3 segments in the example
ns_r  $\leftarrow$  (nr-1)/ $\Delta r$       % 4 segments in the example
If ns_r < ns_c Do
    limited_by_few_rows  $\leftarrow$  true
    mns  $\leftarrow$  ns_r
Else
    limited_by_few_rows  $\leftarrow$  false      % false in the example
    mns  $\leftarrow$  ns_c      % mns  $\leftarrow$  3
End If

rem_r  $\leftarrow$  nr - mns  $\cdot \Delta r$  - 1      % remaining rows  $\leftarrow$  1
rem_c  $\leftarrow$  nc - mns  $\cdot \Delta c$  - 1      % remaining columns  $\leftarrow$  0
nnr  $\leftarrow$  nr      % new number of rows
nnc  $\leftarrow$  nc
 $n\_align_{mns} \leftarrow$  (rem_r+1)  $\cdot$  (rem_c+1)      %  $n\_align(3) \leftarrow$  2
For all cs from mns - 1 to 1 Do
    nnr  $\leftarrow$  nnr - rem_r - 1      % nnr  $\leftarrow$  3 (cs=2) and 2 (cs=1)
    nnc  $\leftarrow$  nnc - rem_c - 1      % nnr  $\leftarrow$  6 (cs=2) and 4 (cs=1)
    If limited_by_few_rows Do
         $n\_align_{cs} \leftarrow$  rem_c  $\cdot$  ( $\Delta r$ -rem_r-1)
    Else
         $n\_align_{cs} \leftarrow$  rem_r  $\cdot$  ( $\Delta c$ -rem_c-1)
        %  $n\_align(2) \leftarrow$  1 and  $n\_align(1) \leftarrow$  0
    End If
    rem_r  $\leftarrow$  nnr - cs  $\cdot \Delta r$  - 1      % 0 (cs=2) and 0 (cs=1)
    rem_c  $\leftarrow$  nnc - cs  $\cdot \Delta c$  - 1      % 1 (cs=2) and 1 (cs=1)
    Increment  $n\_align_{cs}$  By 2  $\cdot$  (rem_r+1)  $\cdot$  (rem_c+1)
    % Finally  $n\_align(2) \leftarrow$  5 and  $n\_align(1) \leftarrow$  4
End For
Return mns and  $n\_align$ 
End Function

```

Appendix A.4. Pseudocode for obtaining annual energy production of a wind farm from the regular alignments

```

Function Energy_Produced
Clear Vector energy
For all the line orientations Do
    Compute d_segment  $\leftarrow$  Distance_of_an_Segment( $\Delta c$ ,  $\Delta r$ )
    For all deviations  $\theta$  from 0 to  $\max_{\beta}(\delta_{lim})$  Do
        wind_direction( $\theta$ )  $\leftarrow$  deviation( $\delta$ ) + line_orientation( $\varphi$ )
        Compute  $fr_{\theta} \leftarrow$  Direction_2_Frequency( $\theta$ )
        For all ws from  $v_{cut\_in}$  to  $v_{cut\_out}$  Do
            Compute  $fr_{ws} \leftarrow$  WindSpeed_2_Frequency(ws)
            For all cdist from 1 to maximum number of segments mns Do
                distance  $\leftarrow$  cdist  $\cdot$  d_segment
                Compute  $deficit_{cdist} \leftarrow$  Wind_Speed_Deficit( $\theta$ , ws, distance)
            End For
            % For each of the "shadowed turbines"
            For all ct2 from 1 to mns Do
                sum_deficit2  $\leftarrow$  0
                % For each of the "shadower turbines"
                For all ct1 from 0 to ct2 - 1 Do
                    cdist  $\leftarrow$  ct2 - ct1
                    deficit2  $\leftarrow$   $deficit_{cdist}$ 
                    Increment sum_deficit2 By deficit2
                End For
                 $v_{ef} \leftarrow$  ws -  $\sqrt{sum\_deficit2}$  % Effective wind speed
                % Generated power at the effective speed
                Compute P  $\leftarrow$  WindSpeed_2_Power( $v_{ef}$ )
                Increment  $energy_{cdist}$  By P  $\cdot$   $fr_{ws} \cdot fr_{\theta} \cdot 8766$ 
            End For
        End For
    End For
End For
% Obtain how many turbines are perturbed by 1 turbine, how many by 2...
% See figure 5
num_turbines_at_posicionmns  $\leftarrow$  n_alignmns
non_perturbed_turbines  $\leftarrow$  nt - n_alignmns
For all ct from mns-1 to 0 Step -1 Do
    num_turbines_at_posicionct  $\leftarrow$  n_alignct + num_turbines_at_posicionct+1
    Decrement non_perturbed_turbines By num_turbines_at_posicionct
End For
% Add up to the total produced energy
Compute energy_of_isolated_turbine From Weibull Distribution and Power Curve
park_energy  $\leftarrow$  non_perturbed_turbines  $\cdot$  energy_of_isolated_turbine
For all ct from 0 to mns Do
    Increment park_energy By num_turbines_at_posicionct  $\cdot$  energyct
End For
Return energy
End Function

```

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