

On the Effects of the Design of Cable-Driven Robots on Kinematics and Dynamics Models Accuracy

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Abstract

This paper tackles the problem of designing and modeling the connection between the fixed frame and end-effector of planar and spatial cable-driven robots. The effect of uncertainties in cables connections on kinematic and dynamic models is investigated. A new proposed design concept is detailed, based on a modification of the end-effector connection consisting in adding pulleys at the end-effector with respect to the usual ways of building cable robots. The main advantage of the proposed design solution is that it does not affect the kinematic and dynamic models, as they are analytically equivalent to the usual point-to-point connection model, but with no pose estimation error owing to the usual simplification of single point/fixed point. An analytical proof of the equivalence of both models is presented. Finally, experimental results are presented for both, planar and spatial cable-robots in order to illustrate the effective advantages of using this novel design concept for connection between the end-effector and fixed frame.

Keywords: cable-driven robot, kinematics model, dynamics model,

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1. Introduction

Cable-driven robots are a class of parallel manipulators [1] that consist of a moving end-effector connected to a fixed frame by means of cables [2, 3]. During the last decade, several planar and spatial versions of cable-driven manipulators have been proposed [4, 5, 6, 7, 8].

In this sense, some works describe the influence of the frame pulleys on the kinematics of cable robots [9], or reduce the positioning error taking them into account in the kinematic model (e.g. [10, 11]).

In all these prototypes and design solutions, cables are commanded by electrical actuators, mainly DC-motor (with or without gearbox) owing to the necessary application of a torque control (e.g. [12]) to properly control the position/orientation of the end-effector [13]. The usual solution for connecting DC-motors and end-effector by means of cables is to use winches (to roll each cable in and out) and pulleys (when a change of the cables direction is needed) [14]. Once the actuation is ready to be fixed to the end-effector, most of the works fix it to the end-effector at a point [3, 4, 5, 6, 7, 8, 13, 14, 15].

More specifically, there are two possible ways to connect a cable to the fixed frame for control issues. The most common design concept used for the fixed frame cable connection is the so-called winch-based system. It is based on the classical cable robot approach using motorized winches, as in [16, 17]. Another approach to be used for the fixed frame cable connection is the use of a rail-based system. Therefore, cables are considered as links of constant length, driven by a skid-rail system. Although in most of the proposed designs of this type [18] each two skids share a common rail, every skid is separately operated by a DC motor through a drive belt. This makes this design solution similar to a linear drive [19].

On the other hand, most of the published works present cable manipulators designs assuming that no sensor is available for a direct measurement of the

end-effector position owing to the fact that this kind of sensors (which provide a
 30 precise real time measurement) are expensive [20]. Therefore, the most extended
 solution consists in estimating the end-effector pose by means of the position of
 the motor/gearbox or pulleys [21, 22], and/or the cable tensions [23]. Hence,
 a very accurate kinematic relation between end-effector position and actuator
 position is needed to estimate the end-effector pose. Therefore, we have to
 35 distinguish between two kinds of approaches:

- a) Works in which the kinematic and dynamic effects of the pulleys are ne-
 glected and they are only taken into account to calculate the relation be-
 tween the motor torques and cable tension, and to estimate cable length
 variations [24, 25]. In this case, although the proposed control strategies
 40 could provide high accurate results in the reference tracking, these model-
 ing assumptions may cause errors in the estimation of the end-effector pose
 (which is used as reference in the control loop) as well as other inherent
 errors related to the kinematic and dynamics.
- b) Works in which the kinematic effects of including the pulleys are assumed
 45 and added to the model [26, 27, 10, 11]. In this case, the reference error
 described in a) is zero, but model complexity considerably increases.

It is usual to find works focused on improving mechanical designs in order to
 reach more accurate models (e.g. [28, 29]). In this paper, a new design concept
 is proposed that overcomes the problems described in a) and b), i.e., providing
 50 kinematic and dynamic models, which allow an accurate relationship between
 end-effector pose and joints angles, but without increasing the complexity of
 such models.

This paper is organized as follows: Section 2 details the conventional model-
 ing for cable robots and the kinematic and dynamic errors that originate. Sec-
 55 tion 3 presents the new design concept for building cable robots, which allows
 using the conventional models without affecting the Kinematics and Dynamics
 estimation. Section 4 supports the novel proposal by means of experimental
 results for both, a planar and a spatial case. Finally, Section 5 summarises the

main conclusions obtained in this work.

60 2. Problem Statement

2.1. Conventional point-to-point model: planar case

This subsection is devoted to detail the conventional point-to-point model for planar cable robots. This model assumes that cables connect the frame (from fixed points placed at pulleys centroid) to the fixed points at the end-effector, neglecting the effects of the pulleys radius in Kinematics and Dynamics equations.

Let's assume a planar n cable robot (see Fig. 1), in which every cable goes from a winch attached to the actuator (Fig. 1 detail 1), then to a pulley (Fig. 1 detail 2), and, finally, to a fixed point at the end-effector (Fig. 1 detail 3). The position of the i th pulley axis fixed to the frame is denoted as $\mathbf{Q}_{fi} = [x_{fi}, y_{fi}]^T$ and the position of the i th joint (connecting cable i and end-effector) as $\mathbf{Q}_{ei} = [x_{ei}, y_{ei}]^T$, with $i = 1, \dots, n$, being n the number of cables. The end-effector pose has been denoted as $\mathbf{Q} = [x, y, \theta]^T$, the i th cable angle as θ_i , each actuator angle as α_i and cable lengths as L_i . Figure 1 illustrates this point-to-point model scheme and some constructive details.

The conventional kinematic and dynamic models assume that cables goes from the pulleys centroid, $[x_{fi}, y_{fi}]$, to the fixed joint points at the end-effector, $[x_{ei}, y_{ei}]$. This model is detailed in the following Sections.

2.1.1. Kinematics

80 The forward kinematics relates actuators angles, $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_n]^T$ and end-effector position $\mathbf{Q} = [x, y, \theta]^T$, and inverse kinematics describes the inverse relation.

Let's denote the centroid of the frame as $[x_c, y_c]^T$ and consider that actuator angles are at home position, $\alpha_i = 0$, when the end-effector centroid is placed at the centroid of the fixed frame with zero orientation angle. Therefore, the initial pose of the end-effector can be written as

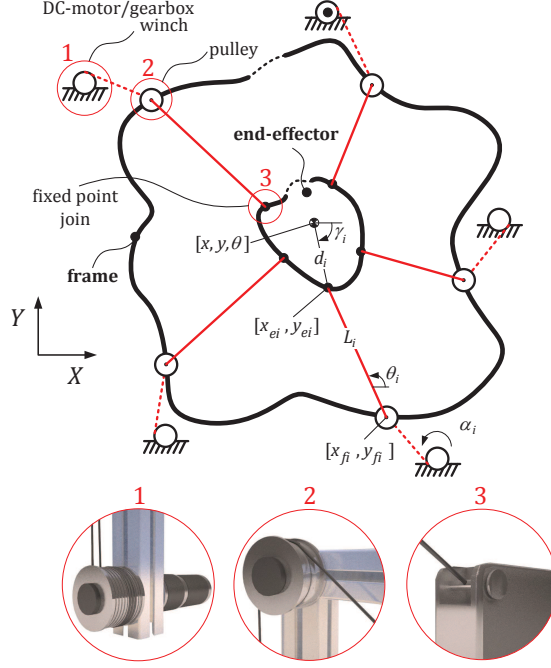


Figure 1: Conventional planar point-to-point model for n cable-driven robot

$$\mathbf{Q}^0 = [x^0, y^0, \theta^0]^T \equiv [x_c, y_c, 0]^T \quad (1)$$

At this configuration, the initial cable lengths, $\mathbf{L}^0 = [L_1^0, L_2^0, \dots, L_n^0]^T$, are

$$L_i^0 = \sqrt{(y_{fi} - y_{ei}^0)^2 + (x_{fi} - x_{ei}^0)^2} \quad (2)$$

where x_{ei}^0 and y_{ei}^0 define the position of the i -th end-effector joint at the initial position, which is known. At this configuration, positive actuators angles, α_i , originate negative changes in the cable lengths, ΔL_i , that can be expressed as $\Delta L_i = -\alpha_i r$, being r the pulley radius. Therefore, actuator angles can be obtained as

$$\boldsymbol{\alpha} = -\frac{1}{r} \Delta \mathbf{L} \quad (3)$$

We can consider that Fig. 1 represents the initial end-effector pose given by (1), in which d_i is the distance from the end-effector centroid to the joint point of cable i th, and γ_i is the corresponding angle at the initial configuration. Therefore, for a given end-effector pose, $\mathbf{Q} = [x, y, \theta]^T$, these joint points are determined by

$$\begin{bmatrix} x_{ei} \\ y_{ei} \end{bmatrix} = \begin{bmatrix} x + d_i \cos(\theta + \gamma_i) \\ y + d_i \sin(\theta + \gamma_i) \end{bmatrix} \quad (4)$$

In this way, the inverse kinematics, $\boldsymbol{\alpha} = \boldsymbol{\Lambda}^I(\mathbf{Q})$, can be determined as

$$\begin{aligned} \alpha_i &= \lambda_i^I(x, y, \theta) = \\ &= \frac{1}{r} \left(L_i^0 - \sqrt{(y_{fi} - y - d_i \sin(\theta + \gamma_i))^2 + (x_{fi} - x - d_i \cos(\theta + \gamma_i))^2} \right) \end{aligned} \quad (5)$$

and the forward kinematics, $\mathbf{Q} = \boldsymbol{\Lambda}^F(\boldsymbol{\alpha})$, by solving the inverse problem of (5), which can be written in a general form as

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} \lambda_x^F(\alpha_1, \alpha_2, \dots, \alpha_n) \\ \lambda_y^F(\alpha_1, \alpha_2, \dots, \alpha_n) \\ \lambda_\theta^F(\alpha_1, \alpha_2, \dots, \alpha_n) \end{bmatrix} \quad (6)$$

2.1.2. Dynamics

The end-effector dynamics can be expressed by

$$\mathbf{M}\ddot{\mathbf{Q}} = \mathbf{F} \quad (7)$$

with $\mathbf{F} = [F_x, F_y, \tau]^T$, being F_x and F_y the Cartesian components of the force applied to the end-effector, τ the torque, and

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (8)$$

105 where m is the end-effector mass and I_{zz} is its rotational inertia calculated at its center of mass.

On the other hand, $\mathbf{F}$ can be computed by means of the equilibrium of the end-effector forces and torque as

$$\mathbf{F} = \begin{bmatrix} F_x \\ F_y \\ \tau \end{bmatrix} = \mathbf{J}_s \mathbf{T} + \mathbf{F}_g \quad (9)$$

where $\mathbf{F}_g = [0, -mg, 0]^T$, $\mathbf{T} = [T_1, \dots, T_n]^T$ is the array of cables tension, and 110 $\mathbf{J}_s$ is the static Jacobian matrix, that yields

$$\mathbf{J}_s = \begin{bmatrix} J_{11} & J_{12} & \dots & J_{1n} \\ J_{21} & J_{22} & \dots & J_{2n} \\ J_{31} & J_{32} & \dots & J_{3n} \end{bmatrix} \quad (10)$$

being

$$\begin{aligned} J_{1i} &= \cos(\theta_i) \\ J_{2i} &= \sin(\theta_i) \\ J_{3i} &= \sin(\theta_i) (x_{ei} - x) - \cos(\theta_i) (y_{ei} - y) \end{aligned} \quad (11)$$

Jacobian terms J_{3i} can be rewritten using (4) yielding

$$J_{3i} = d_i (\sin(\theta_i) \cos(\theta + \gamma_i) - \cos(\theta_i) \sin(\theta + \gamma_i)) \quad (12)$$

Figure 2 shows a scheme of an actuator/winch set. Its Dynamics equation is given by:

$$\mathbf{J} \ddot{\boldsymbol{\alpha}} + \boldsymbol{\nu} \dot{\boldsymbol{\alpha}} + \boldsymbol{\Gamma} = \boldsymbol{\sigma} - r \mathbf{T} \quad (13)$$

115 where $\mathbf{J}$ and $\boldsymbol{\nu}$ are diagonal matrices, composed by the rotational inertia term of each actuator set, J_i , and its respective viscous friction coefficient, ν_i , $\boldsymbol{\Gamma}$ is a diagonal matrix with the non-linear friction terms of each actuator and $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \dots, \sigma_n]^T$, being σ_i the torque exerted by each actuator set.

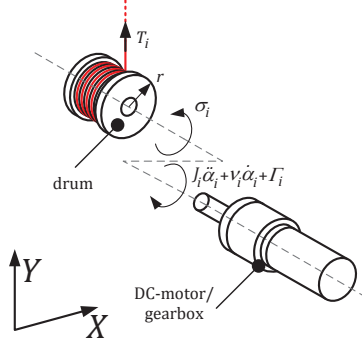


Figure 2: A scheme of the actuator/winch sets.

The complete system dynamic equation relates the system input (torque
 120 exerted by actuators, σ) and the system output (end-effector pose, $\mathbf{Q}$). By
 combining the end-effector dynamics (7), forces and torque equilibrium (9), and
 actuator dynamics (13), the dynamic equation can be written as

$$\sigma = \mathbf{J}_s^{-1} (\mathbf{M} \ddot{\mathbf{Q}} - \mathbf{F}_g) - \mathbf{J} \ddot{\mathbf{A}}^I(\mathbf{Q}) - \nu \dot{\mathbf{A}}^I(\mathbf{Q}) + \mathbf{\Gamma} \quad (14)$$

which can be also expressed related to actuator coordinates, α , as

$$\sigma = r \mathbf{J}_s^{-1} (\mathbf{M} \ddot{\mathbf{A}}^F(\alpha) - \mathbf{F}_g) + \mathbf{J} \ddot{\alpha} + \nu \dot{\alpha} + \mathbf{\Gamma} \quad (15)$$

Dynamics modeling in (14) can be found in e.g. [4, 5, 30, 31].

125 2.2. Spatial case

In this subsection, the above detailed point-to-point model is generalized for
 a spatial cables robot. Let's assume now a 3D scenario where the end-effector
 pose is defined as $\mathbf{Q} = [x, y, z, \varphi_x, \varphi_y, \varphi_z]^T$, being φ_x , φ_y and φ_z the Tait-Bryan
 angles used to express the end-effector orientation [32]. Cables connect the
 130 frame at $[x_{fi}, y_{fi}, z_{fi}]$ to the end-effector at $[x_{ei}, y_{ei}, z_{ei}]$. In the same way, cable
 orientations are expressed in spherical coordinates by means of angles θ_i and
 ϕ_i . Figure 3 illustrates the new 3D scenario.

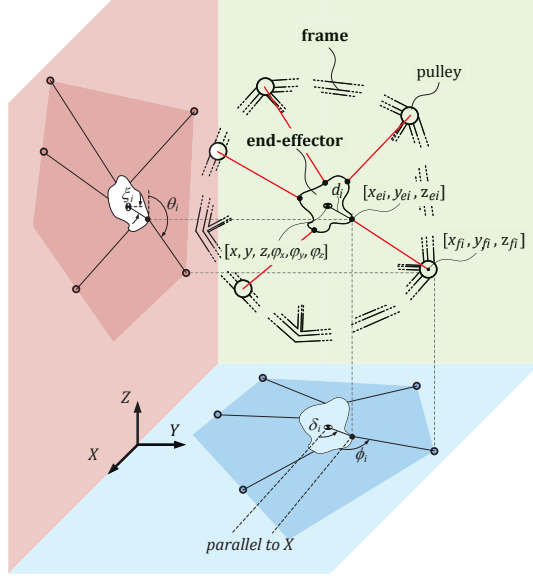


Figure 3: Spatial cable robot: frame/end-effector connection for point-to-point modeling.

Denoting the frame centroid as $[x_c, y_c, z_c]$ and considering that the end-effector initial pose is $\mathbf{Q}^0 = [x_c, y_c, z_c, 0, 0, 0]$ when actuator angles are at home position, $\alpha_i = 0$, then the initial length of each cable is

$$L_i^0 = \left[(x_{fi} - d_i \cos(\xi_i) \sin(\delta_i))^2 + (y_{fi} - d_i \cos(\xi_i) \cos(\delta_i))^2 + (z_{fi} - d_i \sin(\xi_i))^2 \right]^{\frac{1}{2}} \quad (16)$$

The inverse kinematics is denoted as $\boldsymbol{\alpha} = \boldsymbol{\Lambda}^I(\mathbf{Q})$ whereas $\mathbf{Q} = \boldsymbol{\Lambda}^F(\boldsymbol{\alpha})$ is the forward one.

Finally, the static Jacobian matrix, $\mathbf{J}_s$ can be written as

$$\mathbf{J}_s = \begin{bmatrix} J_{11} & J_{12} & \dots & J_{1n} \\ J_{21} & J_{22} & \dots & J_{2n} \\ J_{31} & J_{32} & \dots & J_{3n} \\ J_{41} & J_{42} & \dots & J_{4n} \\ J_{51} & J_{52} & \dots & J_{5n} \\ J_{61} & J_{62} & \dots & J_{6n} \end{bmatrix} \quad (17)$$

where

$$\begin{aligned} J_{1i} &= \cos(\theta_i) \sin(\phi_i) \\ J_{2i} &= \cos(\theta_i) \cos(\phi_i) \\ J_{3i} &= \sin(\theta_i) \\ J_{4i} &= (y_{ei} - y) \sin(\theta_i) \\ &\quad - (z_{ei} - z) \cos(\theta_i) \cos(\phi_i) \\ J_{5i} &= -(x_{ei} - x) \sin(\theta_i) \\ &\quad + (z_{ei} - z) \cos(\theta_i) \sin(\phi_i) \\ J_{6i} &= (x_{ei} - x) \cos(\theta_i) \cos(\phi_i) \\ &\quad - (y_{ei} - y) \cos(\theta_i) \sin(\phi_i) \end{aligned} \quad (18)$$

140 and equations (14) or (15) can be used for modeling the dynamic behavior of spatial cable robots.

2.3. Modeling errors

It is well-known that providing a direct measure of the end-effector pose, $\mathbf{Q}$, to be feedback in the control scheme is expensive [20]. Therefore, the most of the works assume that no sensors are available to provide $\mathbf{Q}$ by direct measure. On the contrary, it is usually estimated by means of (6) and the actuator positions, α , which can be directly measured by means of conventional encoders placed at the motors or at the winches. Therefore, an accurate inverse kinematic transformation (if the control loop is closed using $\mathbf{Q}$) or forward kinematic one (if it is closed with α) is needed [4, 5].

As it was mentioned in Section 2.1, the conventional kinematic and dynamic models assume that cables goes from the pulleys centroid, $[x_{fi}, y_{fi}]$, to the fixed

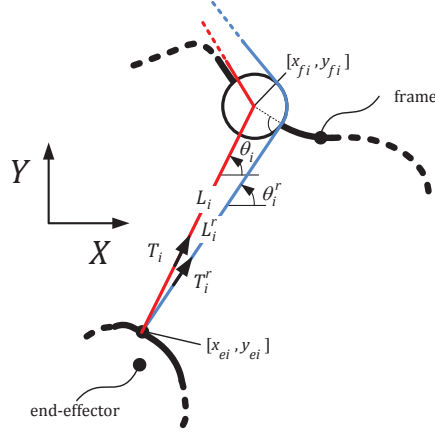


Figure 4: Conventional modeling (red) vs. Real scenario (blue)

joint points at the end-effector, $[x_{ei}, y_{ei}]$. This assumption simplifies the model complexity but originates errors on cable lengths and on cable angles. These errors give rise to inaccuracies in the inverse and forward kinematics, due to the change of the position of the contact point between the cable and the pulley when the end-effector moves (see Fig. 4). In addition, errors in the tension directions (due to errors in the cable orientations) also originates Dynamics uncertainties. Figure 4 illustrates these modeling errors for a planar cable robot, where superindex r denotes the real variables on comparison with the ones used for modeling.

As it was mentioned in the Introduction Section, some works modify the conventional modeling by including the pulley effects in Fig. 4, but obviously the model complexity grows in comparison to the conventional models [26, 27]. The following Section presents a novel mechanical design for connecting cables to the end-effector that allows using the conventional models without falling into the mentioned Kinematics and Dynamics errors.

3. The novel mechanical design solution

3.1. Planar robots

170 In the following, we demonstrate that if the cables that connect the frame
to the end-effector go from a pulley (placed at a fixed point in the frame) to
a pulley (placed at a point of the end-effector) and both, pulley and pulley ,
have the same radius, then the physical system behaves as the point-to-point
model detailed in the previous Section. Figure 5 compares the scheme of the
175 point-to-point model to the proposed one.

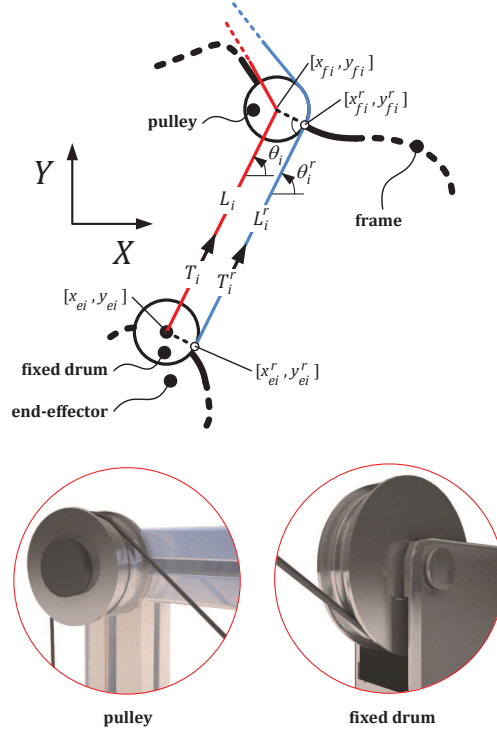


Figure 5: Planar point-to-point scheme (red) vs. proposed scheme (blue).

As it can be seen in Fig 5, the use of a pulley at the end-effector makes that the cable lengths and the cable angles in the real system will be equal to those in the point-to-point model case, $L_i = L_i^r$ and $\theta_i = \theta_i^r$. In this way, the inverse

(5) and forward (6) kinematic transformations, written in the previous section,
 180 can be used for modelling and no estimation errors will be originated.

Taking into account the new contact points, $[x_{ei}^r, y_{ei}^r]$, the static Jacobian matrix of (10) is rewritten as

$$\begin{aligned} J_{1i} &= \cos(\theta_i^r) \\ J_{2i} &= \sin(\theta_i^r) \\ J_{3i} &= \cos(\theta_i^r)(x_{ei}^r - x) + \sin(\theta_i^r)(y_{ei}^r - y) \end{aligned} \quad (19)$$

Note that terms J_{1i} and J_{2i} are exactly equal to (11) owing to $\theta_i = \theta_i^r$. J_{3i} must be rewritten taking into account that now the contact points, $[x_{ei}^r, y_{ei}^r]$,
 185 are

$$\begin{bmatrix} x_{ei}^r \\ y_{ei}^r \end{bmatrix} = \begin{bmatrix} x + d_i \cos(\theta + \gamma_i) + r \sin(\theta_i^r) \\ y + d_i \sin(\theta + \gamma_i) - r \cos(\theta_i^r) \end{bmatrix} \quad (20)$$

Therefore, substituting (20) in (19) and operating, the obtained Jacobian matrix term, J_{3i} , is

$$J_{3i} = d_i (\sin(\theta_i^r) \cos(\theta + \gamma_i) - \cos(\theta_i^r) \sin(\theta + \gamma_i)) \quad (21)$$

that is similar to the obtained with the point-to-point model (12).

Therefore, if the inverse kinematics transformation, $\mathbf{\Lambda}^I$, and the forward one,
 190 $\mathbf{\Lambda}^F$, are equivalent to the point-to-point model ones, and the static Jacobian matrix, $\mathbf{J}_s$, is also equivalent, then the point-to-point dynamic models (14) and (15) can be used for the new proposed cable robot if appropriate pulleys are used at the end-effector, and no errors on the Kinematics or on the Dynamics are originated.

195 3.2. Spatial robots

In an analogous way that in the previous Section, we assume that the end-effector is designed in such a way that the cables that connect the frame to the end-effector go from a pulley (placed at a fixed point in the frame with free ϕ_i rotation) to a drum (placed at a point of the end-effector with free ϕ_i θ_i)

200 and both, pulley and pulley , have the same radius. In these conditions, we will demonstrate that the real system behavior and the point-to-point model detailed in the previous Section are equivalent. Figure 6 compares the point-to-point model scheme to the proposed one.

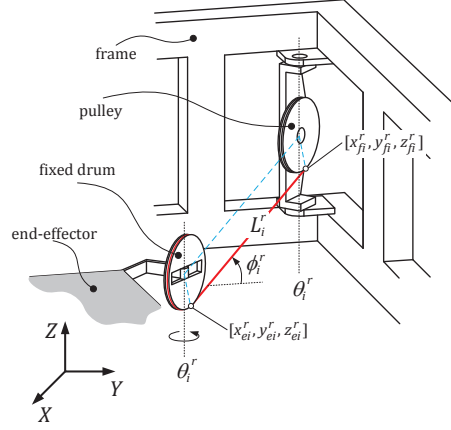


Figure 6: Planar point-to-point scheme (red) vs. proposed scheme (blue).

This way of connecting the frame and the effector respects the cable orientations ($\theta_i^r = \theta_i$ and $\phi_i^r = \phi_i$) and the cable lengths ($L_i^r = L_i$), and therefore
 205 the inverse and forward kinematic transformations, $\mathbf{\Lambda}^I$ and $\mathbf{\Lambda}^F$, are equivalent to the ones obtained in the previous Section for the point-to-point model.

On the other hand, the static Jacobian matrix can be written as (17), where terms $J_{1i} = \cos(\theta_{1i}^r) \sin(\phi_i^r)$, $J_{i2} = \cos(\theta_{1i}^r) \cos(\phi_i^r)$ and $J_{i3} = \sin(\theta_{1i}^r)$ are similar
 210 to those of the point-to-point model case since $\theta_i^r = \theta_i$. Terms J_{4i} , J_{5i} and J_{6i} are like in (18) but replacing, additionally, x_{ei} , y_{ei} and z_{ei} by x_{ei}^r , y_{ei}^r and z_{ei}^r . These contact points yield

$$\begin{bmatrix} x_{ei}^r \\ y_{ei}^r \\ z_{ei}^r \end{bmatrix} = \begin{bmatrix} x_{ei} + r \sin(\theta_i^r) \sin(\phi_i^r) \\ y_{ei} + r \sin(\theta_i^r) \cos(\phi_i^r) \\ z_{ei} - r \cos(\theta_i^r) \end{bmatrix} \quad (22)$$

Replacing (22) in J_{4i} , J_{5i} and J_{6i} terms of (18), they result

$$\begin{aligned}
J_{4i} &= (y_{ei} - y) \sin(\theta_i^r) \\
&\quad - (z_{ei} - z) \cos(\theta_i^r) \cos(\phi_i^r) \\
J_{5i} &= -(x_{ei} - x) \sin(\theta_i^r) \\
&\quad + (z_{ei} - z) \cos(\theta_i^r) \sin(\phi_i^r) \\
J_{6i} &= (x_{ei} - x) \cos(\theta_i^r) \cos(\phi_i^r) \\
&\quad - (y_{ei} - y) \cos(\theta_i^r) \sin(\phi_i^r)
\end{aligned} \tag{23}$$

that are equals to the point-to-point model ones due to $\theta_i^r = \theta_i$ and $\phi_i^r = \phi_i$.

215 In the same way that in the planar case, inverse and forward kinematic transformations, $\mathbf{\Lambda}^I$ and $\mathbf{\Lambda}^F$ are equivalent to the point-to-point model ones, and static Jacobian matrix, $\mathbf{J}_s$, is also equivalent. Therefore, the point-to-point dynamic models (14) and (15) can be used in the new proposed spatial cable robot using pulleys at the end-effector, and no errors on the kinematics or on
220 the dynamics will be originated.

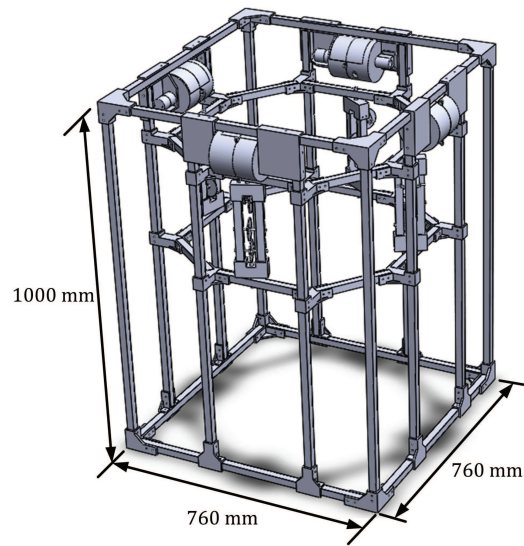
4. Experimental results

In this section the positioning error committed when using the new proposal for connecting the frame (from pulley) to the end-effector (to a pulley) is experimentally obtained. The first subsection presents the experimental platform
225 which allows us to make planar and spatial experiments. Finally, the second and the third sections present the planar and the spatial results, respectively.

4.1. Experimental platform

The prototype has been designed with *Solidworks*® and built with aluminium profiles and 3D printed plastic parts (*PLA*). The final design and prototype are shown in Fig. 7.
230

The overall dimensions of the prototype are $760 \times 760 \times 1000$ mm. Four stepper motors (*NEMA 17*), move four mechanisms which allow rolling in and out the cables over the drums. This roller mechanism makes the exit point of cables remain invariant when the drum rotates (see Fig. 8a). Figures 8b and



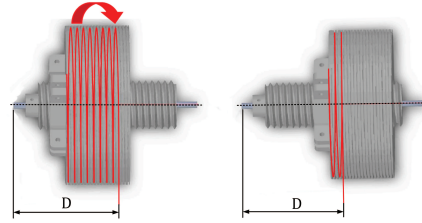
(a) Solidworks design



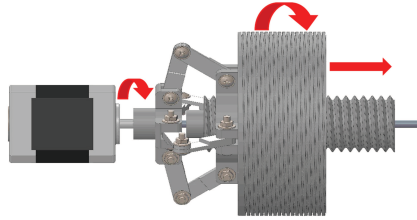
(b) Real prototype

Figure 7: Experimental platform.

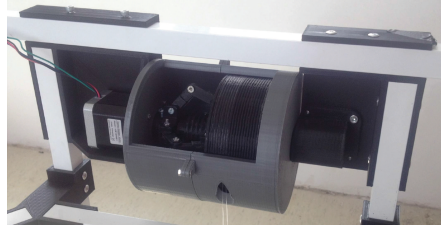
235 8c show the actuator design and the final aspect of the roller mechanism. Note that each mechanism rolls a pair of cables.



(a) Maintaining exit point of cables



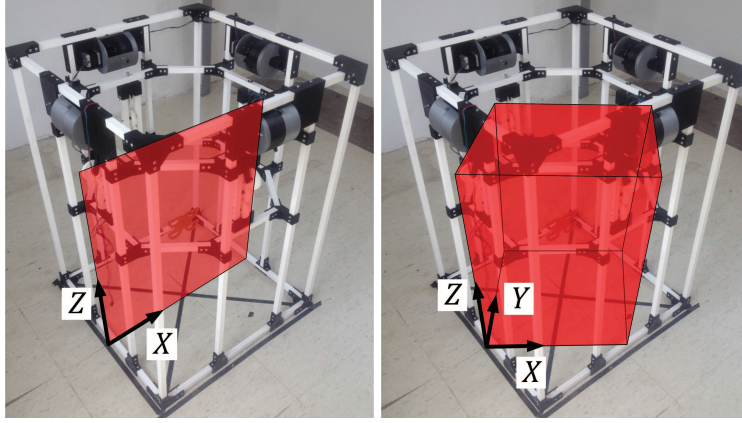
(b) Actuation design



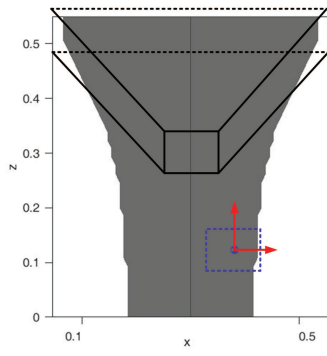
(c) Real actuator

Figure 8: Roller mechanism

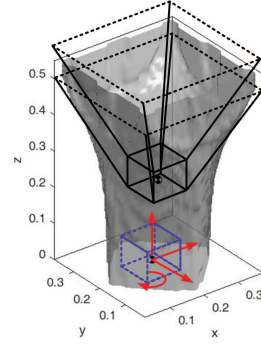
The designed manipulator can be configured to be used as a planar one (two motors with two pairs of cables), or as an spatial one (four motors with four pairs of cables). In the planar case, the end-effector has two degrees-of-freedom, whereas in the spatial case, it has four degrees-of-freedom. Figure 9a and 9b detail both configurations and their degrees of freedom. In both cases, the workspace of the robot can be obtained by assuming an end-effector pose and checking if all cable tensions remains positive (see [14] for more details).



(a) Planar configuration (2 motors) (b) Spatial configuration (4 motors)



(c) Workspace (planar)



(d) Workspace (spatial)

Figure 9: Planar and spatial configuration.

4.2. Case 1: Two DoF planar cable robot

4.2.1. Description

The scheme and notation of the robot is presented in Fig. 10. Note that the end-effector pose is $Q = [x, y]^T$ and this pose is controlled by changing cable lengths. Each pair of cables (the red ones and the green ones) is controlled by the roller mechanism which is actuated by means of the stepper motor described above. In this way, the end-effector rotation is constrained. Denoting motor angles as α_1 and α_2 , then the inverse kinematics (5) is solved in order to control the end-effector pose i.e., $[\alpha_1, \alpha_2] = \mathbf{\Lambda}^I(x, y)$. As previously stated, the radius of the pulleys placed at the frame and the fixed ones at the end-effector are equals, r .

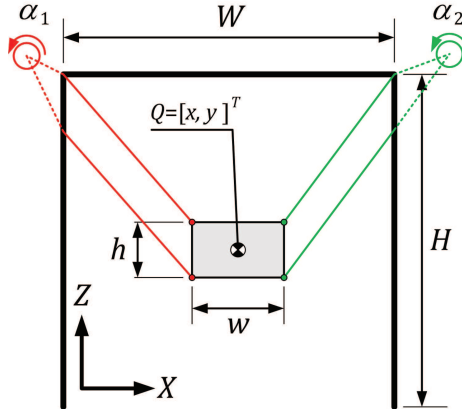


Figure 10: Scheme of the planar manipulator

For this planar configuration two different end-effectors have been compared: one with pulleys, and another one without them, in order to verify the decreasing of the positioning error if pulleys are included at the end-effector connection. Figure 11 shows both end-effectors.

The main parameters of the robot are summarised in Table 1.

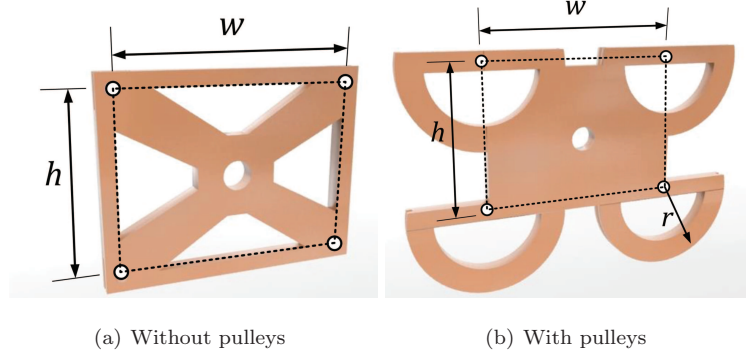


Figure 11: Compared end-effectors

Table 1: Parameters of the manipulator with planar configuration

Symbol	Description	Value
H	height of the frame	600 mm
W	width of the frame	566 mm
h	height of the end-effector	60 mm
w	width of the end-effector	80 mm
m	mass of the end-effector	112 g
r	pulleys/drum radius	30 mm

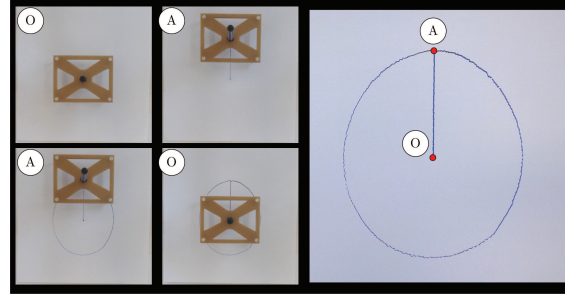
260 4.2.2. Trajectories

In order to compare the accuracy of the point-to-point kinematic model using a conventional point joint end-effector (Fig. 11a) and the proposed one (Fig. 11b), two planar trajectories have been tested for the end-effector positioning: a circumference (100 mm of diameter) and a rhombus (50 mm of side).

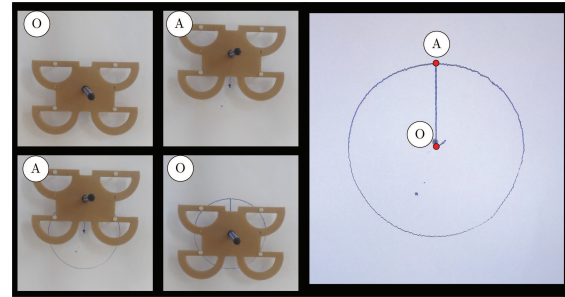
265 The time profile of both trajectories are a 4th order Bezier curve to ensure the differentiability of their velocity and acceleration. The center of both trajectories has been placed at the center of the workspace. The duration is 10 s.

Figure 12 shows the sequence of movements of the circle trajectory for the compared end-effectors, and Fig. 13 represents the sequence for the rhombus

270 trajectory.



(a) Without drums



(b) With pulleys

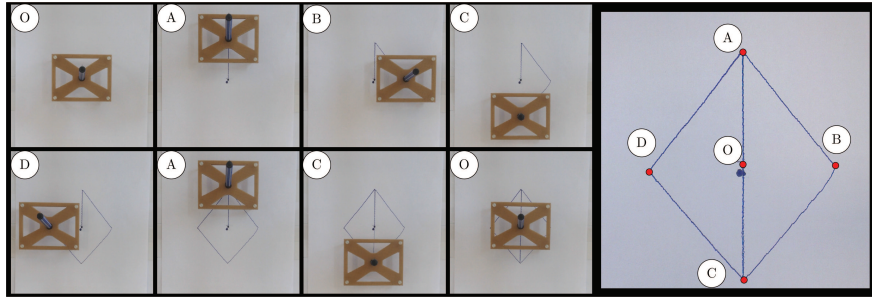
Figure 12: Circular trajectory.

4.2.3. Results

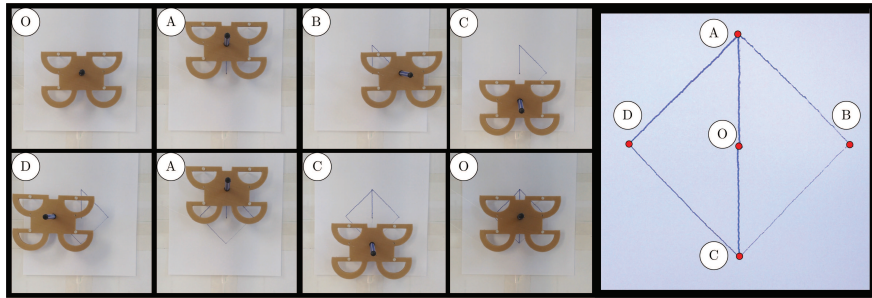
Using *MATLAB Image Toolbox*® it is possible to obtain the image binarization and compare the real trajectory to the reference one for both cases. Owing to the resolution of the camera used each image pixel corresponds to 0.085 mm approximately.

Figure 14 compares the executed trajectories that have been obtained by attaching a conventional pencil to the end-effector. Figure 15 shows the same paths after the image treatment and the reference trajectory.

Defining the committed error in the normal direction to the reference trajectory, it is possible to obtain the number of pixels from the desired reference to the real trajectory and therefore to obtain the tracking error. The compared



(a) Without drums



(b) With pulleys

Figure 13: Rhombus trajectory.

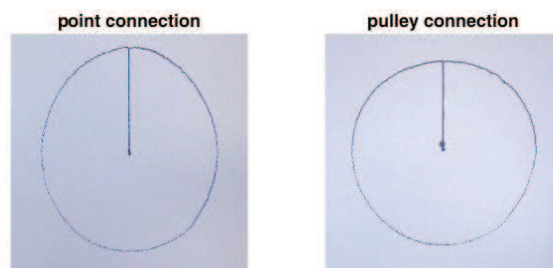


Figure 14: Circle trajectory comparison: original results

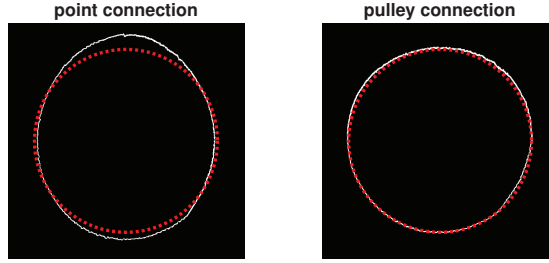


Figure 15: Circle trajectory comparison: image treatment and reference

results are shown in Fig. 16.

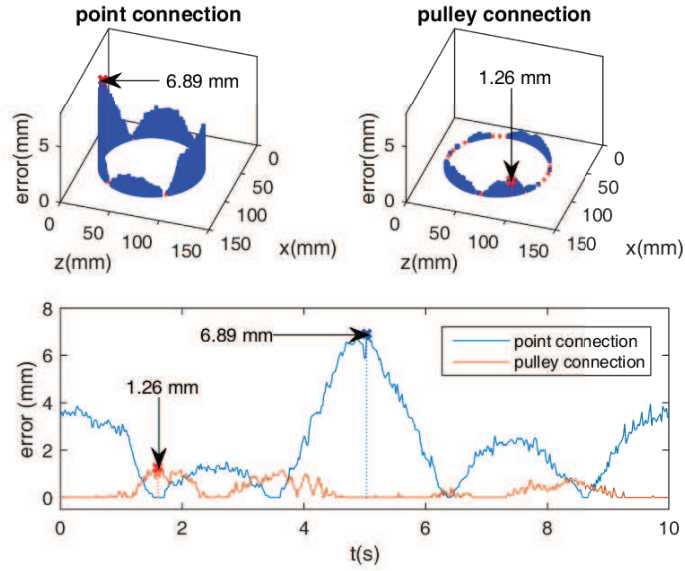


Figure 16: Circle trajectory comparison

The results show that with the convention point connection the maximum tracking error is 6.89 mm (and the average one is 2.24 mm), whereas with the proposed pulley connection, the maximum tracking error is 1.26 mm (and the average one is 0.28 mm).

Finally, as for the rhombus trajectory, Fig. 17 compares the executed tra-

jectories for both end-effectors. Figure 18 shows the same paths after the image
treatment and the reference trajectory and Fig. 19 represents the error com-
mitted for both compared connection ways.

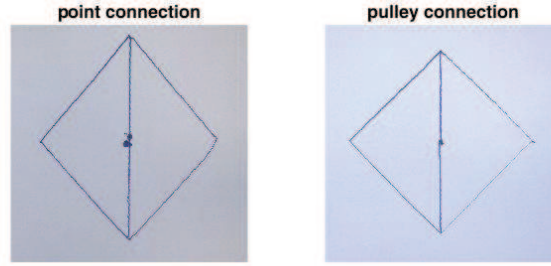


Figure 17: Circle trajectory comparison: original results

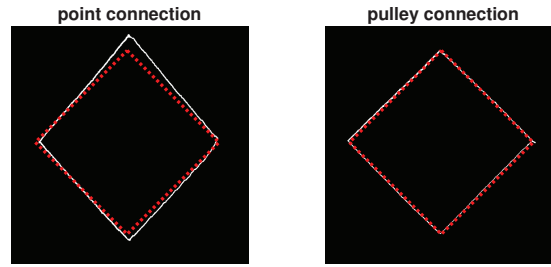


Figure 18: Circle trajectory comparison: image treatment and reference

The results show again a notorious decreasing in the positioning error in
comparison to the conventional point connection. The maximum tracking error
for the conventional point connection is 9.00 mm (and the average one is 2.01
mm). With the proposed pulley connection, the maximum tracking error is 0.76
mm (and the average one is 0.17 mm).

Therefore, adding pulleys to the end-effector (with the same radius as the
frame pulleys) notoriously decreases the positioning error with regard to the
conventional point connection end-effector. It is worth to note that the error
committed using the conventional point connection at the end-effector can be
also reduced by software compensation (e.g. [5]) or taking into account the

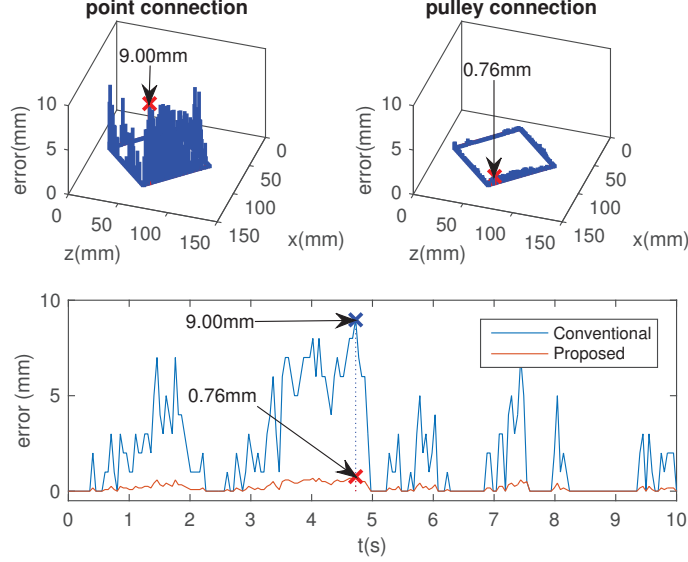


Figure 19: Circle trajectory comparison: committed error

radius of the pulleys in the kinematic model (e.g. [10, 11]). However, the proposed solution provides an easy way to use the conventional point-to-point kinematic model avoiding the errors obtained when the pulley radii are not taken into account in the kinematic model ([9]).

4.3. Case 2: Four DoF spatial cable robot

4.3.1. Description

The scheme and notation of the robot is presented in Fig. 20. Note that the end-effector pose is $Q = [x, y, z, \delta]^T$ and this pose is again controlled by changing the cable lengths. In this case, the four pairs of cables (the red, green, blue and yellow ones) are controlled by the four roller mechanism which are also actuated by the stepper motors. In this way, the end-effector rotation along Y and X axis are constrained. Denoting motor angles as α_1 to α_4 , the inverse kinematics (5) is solved in order to control the end-effector pose i.e., $[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = \mathbf{\Lambda}^I(x, y, z, \delta)$. The radius of the pulleys placed at the frame and the fixed ones at the end-effector are equals, r .

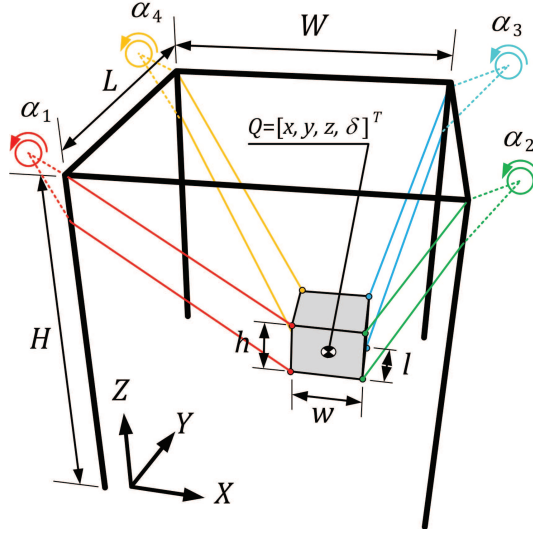


Figure 20: Scheme of the spatial manipulator

Figure 21 shows the designed end-effector.

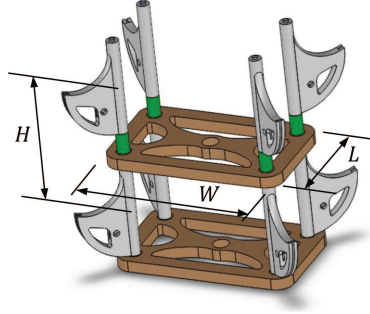


Figure 21: Spatial manipulator: end-effector

The main parameters of the spatial robot are summarised in Table 2.

4.3.2. Trajectories

In this spatial case, only the new pulley connection will be checked. The
 320 trajectories are a circumference (150 mm of diameter) and a rhombus (75 mm
 of side), both executed into the XY plane.

Table 2: Parameters of the manipulator with spatial configuration.

Symbol	Description	Value
H	height of the frame	600 mm
W	width of the frame	400 mm
L	length of the frame	400 mm
h	height of the end-effector	60 mm
w	width of the end-effector	90 mm
l	width of the end-effector	45 mm
m	mass of the end-effector	248 g
r	pulleys/drum radius	30 mm

The time profile for both trajectories are again a 4th order Bezier curve and the center of both trajectories has been placed at the center of the workspace. The duration is 10 s.

325 Figure 22 shows the sequence of movements of the circular trajectory for the compared end-effectors, and Fig. 23 represents the sequence for the rhombus trajectory.

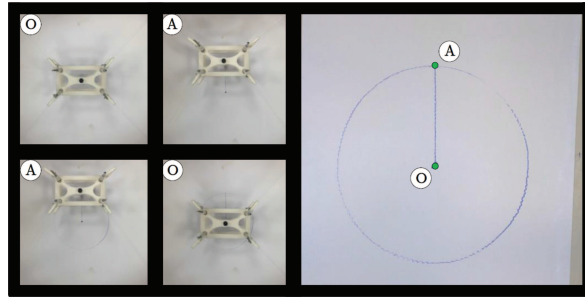


Figure 22: Circle trajectory

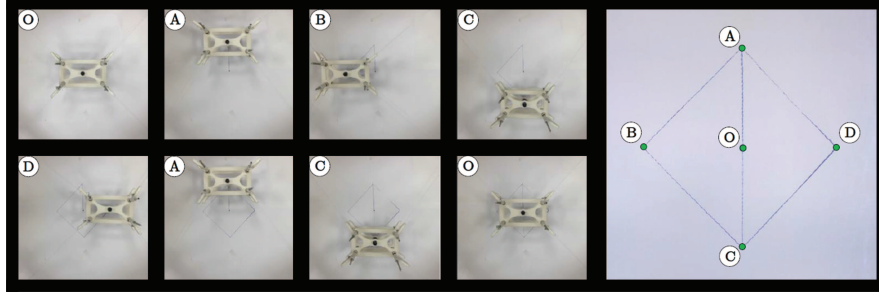


Figure 23: Rhombus trajectory

4.3.3. Results

Using an analogous procedure to that of the previous subsection (*MATLAB*
 330 *Image Toolbox*®) the error of both trajectories has been obtained.

Figure 24 shows the executed trajectories, and Fig. 25 represents the same path after the image treatment and the reference trajectory.

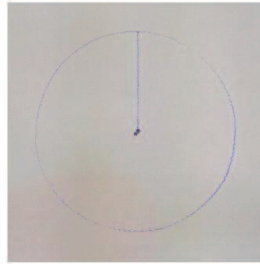


Figure 24: Circle trajectory

The tracking error between the reference trajectory and the real one is shown in Fig. 26.

335 The results show that the proposed frame/end-effector connection provides low values of positioning errors. For the circular trajectory, the maximum tracking error is 1.64mm and its average one is 0.44 mm.

Finally, as for the rhombus trajectory, Fig. 27 compares the executed trajectories for both end-effectors, Fig. 28 shows the same paths after the image
 340 treatment and the reference trajectory, and Fig. 29 represents the error com-

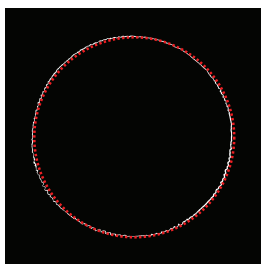


Figure 25: Circle trajectory after image treatment and reference

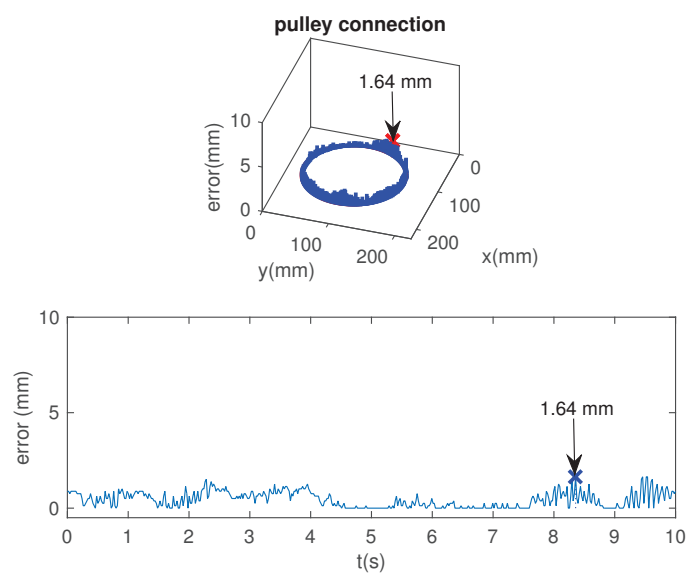


Figure 26: Circle trajectory error

mitted for both compared connection ways.

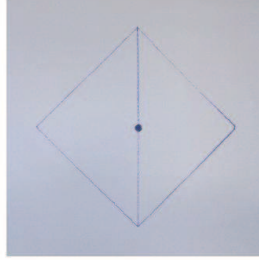


Figure 27: Rhombus trajectory

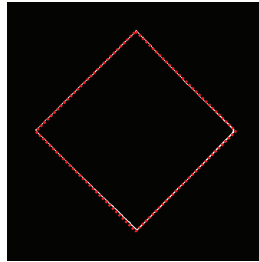


Figure 28: Rhombus trajectory after image treatment and reference

The results show again a low value of the positioning error. The maximum tracking error for the conventional point connection is 1.01 mm and its average one is 0.46 mm.

345 Therefore, as in the planar case, the spatial results support the idea that the proposed solution provides an easy way for using the conventional point-to-point kinematic model while avoiding the errors obtained when the radius of the pulleys is not taken into account.

5. Conclusions and future works

350 In this paper, a new design solution is proposed for connecting the fixed frame to the end-effector for cable robot.

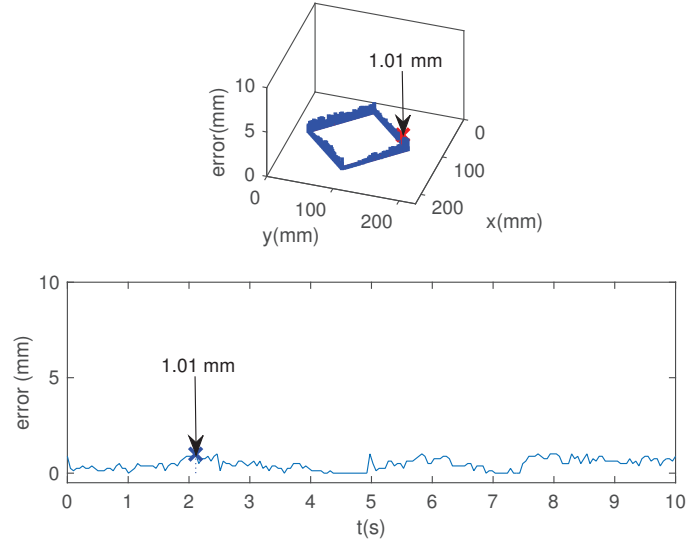


Figure 29: Rhombus trajectory error

Most of the existing prototypes of cable robots are built assuming that no sensor for a direct measurement of the end-effector pose is available owing to that this kind of sensors are nowadays expensive. Therefore, the most extended solution consists in estimating the end-effector pose by means of the motor/gearbox sets or the pulleys positions. In this way, an accurate kinematic relation between end-effector pose and actuators positions results necessary.

The conventional point-to-point model used to describe dynamic behavior of cable robots has been used for both planar and spatial cables robots. This usual way of modeling assumes that no pulleys are placed at the fixed frame, and the radius is only taken into account to compute the cable length variations when drums roll them in or out. Indeed, this paper shows that some inherent errors appears when this simplification is made due to the difference between model and real system cable lengths and orientations, that in turn originates dynamics behavior differences.

In order to overcome the above-mentioned problems, we propose a novel

design solution assuming the use of pulleys at the end-effector, with the same radius than the drums or/and pulleys attached to the frame. This solution makes the cable lengths and orientations equivalent to those of the point-to-point model. In addition, the static Jacobian matrix is also analytically equivalent to the point-to-point model one and then, the usual model (14) can be used to describe the system dynamics behavior while avoiding the above mentioned errors.

Finally, two application examples that support the proposal have been presented: a two DoF cable manipulator and a four DoF spatial one. In both cases, by using the point-to-point conventional model, the maximum positioning error committed is about 1 mm.

Therefore, adding pulleys to the end-effector allows using the conventional and simple point-to-point kinematic model, while avoiding the loss of accuracy derived from disregarding the radius of the pulleys in the kinematic model (as is detailed in [9]). This proposal can be considered as an alternative way to the actual techniques consisting in software compensation (e.g. [5]) or taking into account the radius of the pulleys in the kinematic model (e.g. [10, 11])

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