

Improving the energy efficiency and speed of walking robots

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Abstract This study analyses the superior performance of gravitationally decoupled actuation in terms of energy efficiency, and hence autonomy. Based on the decoupling concept, a new design is presented for a 3 dof leg, and its performance is validated by including it in a 84kg hybrid locomotion robot. The proposed leg obtains a straight-line constant-velocity motion of the foot as only one of its 3 motors is operated at constant speed. This feature, together with the hybrid structure, increases the robot's efficiency and speed when operating on surfaces without obstacles, and drastically simplifies the walking operation control. A series of simulations have been done comparing the energy consumption of the hybrid robot including traditional coupled legs, and including the proposed decoupled legs. They show a superior performance of the later one. A real prototype has been built including the proposed mechanism. It shows a similar performance to that obtained from simulation, and a natural gait in flat terrains, at speeds up to 0.9 m/s.

Keywords legged locomotion · hybrid locomotion · kinematics · simulation · energetic efficiency · minimum energy control · Petri nets

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1 Introduction

Legged robots or, in general, walking machines have gained growing interest over recent years due to their superior terrain adaptability. They can move on soft or slippery terrains and overcome obstacles. However, their main drawback is their reduced autonomy derived from low energy efficiency and scarce load capacity. Most of the existing robots make an incomplete mimic of the human/animal leg structure, based on sequencing links and joints. However, these simple mechanical structures impede the accomplishment of easy movements without complex control operations and, what is more important, they lack an energetic optimization study, which greatly reduces their autonomy.

Since existing technology cannot achieve a complete mimic of the locomotion structure of a medium-to-large animal, a deeper study must be done based on existing resources and released from partial copies of real animal legs, mainly when they correspond to animals with different characteristics (habitat, weight or capacities) to those assumed for the designed robot. For this purpose, some ideas have arisen in the fields of walking and hybrid robots.

The most successful ones come from studies made over biped robots, mainly the ones that develop the concept of Passive Dynamic Walkers (PDW), introduced by McGeer (1990a,b). Collins et al (2005) show additional examples of PDW. These ideas have promoted PDW to be the most efficient walking robots measured in terms of their *specific resistance*. This term was first introduced by Gabrielli and von Karman (1950) as a figure to represent the energy efficiency during locomotion and, with regard to walking robots, it is obtained

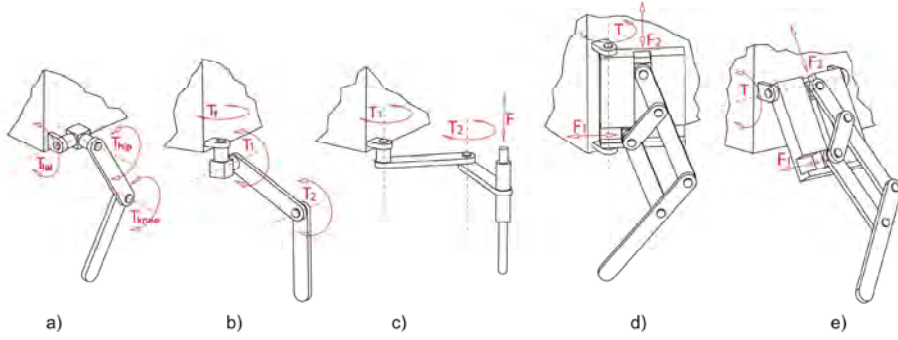


Fig. 1 Different types of leg: a) GC-VC legs; b) coupled GC-LC legs; c) SCARA-like leg (GD-LC); d) GD-LC leg (2D pantographic); e) GD-VC leg (2D pantographic).

from

$$\epsilon = \frac{E}{M g d} \quad (1)$$

where E is the total energy consumption for a distance travelled d , and Mg is the robot weight. Jin et al (2013) have analyzed the effect of several parameters (mass distribution, stride length, duty factor, walking velocity and height) on the specific resistance of an hexapod walking robot.

This term relies on the same idea as the *cost of transport*, defined by Collins et al (2005), which is preferable in the case of robots, because it distinguishes between the mechanical cost of transport, c_{mt} (easier to calculate from simulations), and the electrical cost of transport c_{et} . This figure is analogous to specific resistance, easier to calculate from real experiments and more useful for battery dimensioning.

In the case of hybrid robots and walking robots with more than two legs, the prototypes have not achieved the same degree of energy efficiency. The main reason is that the legs included in most of the robot designs are gravitationally coupled (GC) as defined by Hirose et al (1991, 2009) and Kar (2003). It means that during the support phase, there is at least one actuator that, at the same time, is moving and withstanding the weight. On the contrary, in gravitationally decoupled (GD) legs, the moving actuators do not bear the weight, and theoretically, no power is expected during the stance phase of GD legs due to the robot weight. Since the support phase is the part of the walking cycle demanding a higher energy consumption, GD legs success in improving the efficiency and autonomy if the system cannot regenerate negative power, i.e. if the system cannot transform the kinetic energy into electrical one. In terms of efficiency, the GD configurations give rise to an improved performance in comparison to GC ones. Added to this assert, Hirose et al. and Kar outline that

GD actuation allows a more adequate selection of actuators. This leads to motors with lower rated capacity, and hence lower weight.

Another issue that determines the robot characteristics is the orientation in which the leg is connected to the hip. This way the leg can be laterally connected (LC), as in reptiles or crabs, or vertically connected (VC), as in mammals.

According to these issues, a classification of the main types of leg structures is presented in figure 1, which is similar to that exposed by Kar (2003).

The first type of legs in this figure, GC-VC legs, is the preferred one in modern robots, since this type mimics the mammal configuration. The second type is also very popular in walking robots, and it is included in many of the Titan series, the Alduro or the Silo series (Silo6 can also adopt a pseudo-mammal configuration). The Scara-like leg is used in climbing robots as ROWER or REST. The Roboclimber adopts a cylindrical configuration, which is not included in figure 1. The pantographic solutions were very popular in past since these legs can be controlled in an easier and faster way. The Turtle of Adachi et al (1988) or the ASV of Waldron and McGhee (1986) belongs respectively to GD-LC legs and GD-VC leg.

This paper presents a comparison in terms of consumed energy and speed between GC legs and GD legs. In section 2 analytical results are obtained for two types of GC legs along the support phase, which is the more determining phase in terms of energy. This section also includes the performance description of the proposed GD-VC leg, which performs a quasi-Cartesian movement. Therefore, the foot can be forwardly moved at constant speed by moving one joint at constant speed.

Section 3 adds simulation results to this reasoning, comparing the performance of a hybrid robot including GC-VC legs and the same hybrid robot including the proposed leg.

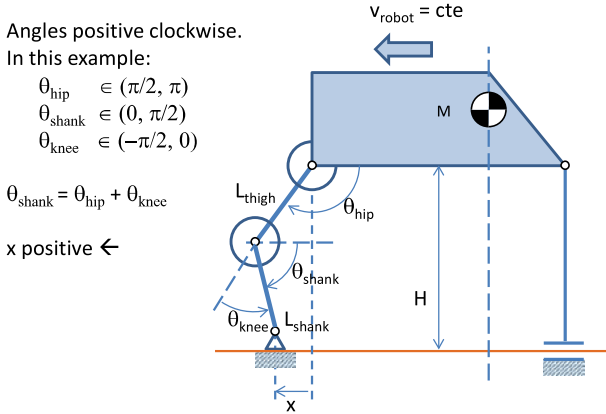


Fig. 2 Representation of a hybrid locomotion robot equipped with GC-VC legs, during the support phase.

Section 4 presents a real hybrid prototype with the proposed decoupled legs.

Finally, section 5 resumes the goals of the paper, outlining the main conclusions.

2 Power losses of gravitationally coupled schemes

The study presented in this section analyses from a theoretical point of view the power losses during the support phase of two types of GC legs, and it will show that these legs waste a significant amount of energy without increasing the mechanical energy. These losses are not present in decoupled configurations, [as the one presented at the end of the section](#).

2.1 Losses in mammal-like (GC-VC) legs

The proposed leg is part of a hybrid robot, to be presented in sections 3 and 4. In order to establish a proper energy comparison to the proposed leg, we will assume that a GC-VC leg (subfigure 1(a)) is also included as a part of a hybrid robot, walking at velocity v_{robot} , with mass M , and rated leg stroke S . The thigh and shank lengths are denoted as L_{thigh} and L_{shank} , respectively, and the height from ground to hip is obtained from these three lengths, as H . By assuming that the hip is located at $(0, H)$, the following equations represent the joint angles (θ_{hip} and θ_{knee}) of GC-VC leg with rotational joints as a function of the foot position x during the support phase (see figure 2):

$$\begin{aligned}L_{thigh} \sin \theta_{hip} + L_{shank} \sin \theta_{shank} &= H \\ \text{with } \theta_{shank} &= \theta_{hip} + \theta_{knee} \\ L_{thigh} \cos \theta_{hip} + L_{shank} \cos \theta_{shank} &= x\end{aligned}\quad (2)$$

Robot speed	$v_{robot} = 0.6 \text{ m/s}$
Rated step	$S = 0.58 \text{ m}$
Thigh length	$L_{thigh} = 0.522 \text{ m}$
Shank length	$L_{shank} = 0.522 \text{ m}$
Thigh weight	$M_{thigh} = 1 \text{ kg}$
Shank weight	$M_{shank} = 1 \text{ kg}$
Height	$H = 1 \text{ m}$
Inertia	$I = ML^2/12$

Table 1 Mammal-like (GC-VC) leg. Data used for the simulation of the stance phase.

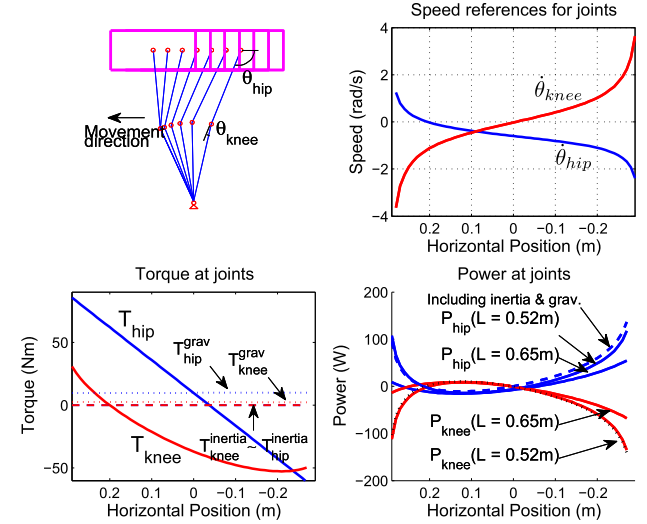


Fig. 3 Performance of a coupled mammal-like leg during the support phase: leg and body evolution, speed references, torques and power for the joints at the hip and knee.

The solution is

$$\theta_{shank} = \arccos \left(\frac{H^2 + L_{shank}^2 + x^2 - L_{thigh}^2}{2L_{shank}\sqrt{x^2 + H^2}} \right) + \gamma \quad \text{with}$$

$$\gamma = \arctan \left(\frac{H}{x} \right) \quad \gamma \in (-\pi/2, \pi/2)$$

$$\theta_{hip} = \arccos \left(\frac{x - L_{shank} \cos(\theta_{shank})}{L_{thigh}} \right) \quad (3)$$

$$\theta_{knee} = \theta_{shank} - \theta_{hip}$$

The joint speed references ($\dot{\theta}_{hip}$ and $\dot{\theta}_{knee}$) are obtained from $\dot{x} = -v_{robot}$ (see figure 3), applying the inverse of the Jacobian

$$\dot{\Theta} = J^{-1}V \quad (4)$$

with

$$\begin{aligned}\dot{\Theta} &= \begin{bmatrix} \dot{\theta}_{hip} \\ \dot{\theta}_{knee} \end{bmatrix} \quad V = \begin{bmatrix} -v_{robot} \\ 0 \end{bmatrix} \\ J^{-1} &= \frac{\begin{bmatrix} -L_{shank}cs & -L_{shank}ss \\ L_{thigh}ch + L_{shank}cs & L_{thigh}sh + L_{shank}ss \end{bmatrix}}{L_{thigh}L_{shank}sk}\end{aligned}\quad (5)$$

and where, for sake of compactness, we have substituted

$$\begin{aligned} ch &= \cos(\theta_{hip}) & sh &= \sin(\theta_{hip}) \\ ck &= \cos(\theta_{knee}) & sk &= \sin(\theta_{knee}) \\ cs &= \cos(\theta_{shank}) & ss &= \sin(\theta_{shank}). \end{aligned} \quad (6)$$

With regard to the torque, for a hybrid or quadruped robot, each one of the three supports (legs or wheels) must withstand **half of the weight**, and if friction is disregarded, then

$$T_{hip} = \frac{Mg}{2}x + T_{hip}^{grav+inert} \quad (7)$$

$$T_{knee} = \frac{Mg}{2}(x - L_{thigh} \cos \theta_{hip}) + T_{knee}^{grav+inert} \quad (8)$$

where the additional addends are the torque due to inertia and the weight of thigh and shank. Lagrange formulations provides these values:

$$\begin{aligned} A &= \frac{1}{3}M_{thigh} L_{thigh}^2 + M_{shank}L_{thigh}^2 \\ B &= \frac{1}{3}M_{shank} L_{shank}^2 \\ C &= \frac{1}{2}M_{shank}L_{thigh}L_{shank} \end{aligned} \quad (9)$$

$$\begin{aligned} T_{hip}^{inertia} &= \ddot{\theta}_{hip}(A + B + 2Cck) + \ddot{\theta}_{knee}(B + Cck) + \\ &\quad -(2\dot{\theta}_{hip} + \dot{\theta}_{knee})\dot{\theta}_{knee}Csk \end{aligned} \quad (10)$$

$$T_{hip}^{grav} = M_{thigh}g \frac{L_1}{2}sh + M_{shank}g \left(L_t sh + \frac{L_{shank}}{2}ss \right)$$

$$\begin{aligned} T_{knee}^{inertia} &= \ddot{\theta}_{hip}(B + Cck) + \ddot{\theta}_{knee}B + Csk\dot{\theta}_{hip}\dot{\theta}_{shank} + \\ &\quad -\dot{\theta}_{hip}\dot{\theta}_{knee}Csk \end{aligned} \quad (11)$$

$$T_{knee}^{grav} = M_{shank}g \frac{L_s}{2}ss.$$

Table 1 lists the data used for the simulations, and Figure 3 presents the simulation results. As can be seen from the torque plot, inertia term can be neglected during the stance phase. Power plot (lower right-hand plot) includes the simulation result for a longer leg, with $L_{thigh} = L_{shank} = 0.65m$. This plot shows three additional issues:

- Regardless of the significant value of the gravitational term of torque, the effect of this term over the power is neither to be considered. To this end, dashed curves have been included, which refer to the power supplied to the joints taking into account the torque due to inertia and the leg weight (for $L_{thigh} = L_{shank} = 0.522m$).

- Disregarding inertia and gravitational terms of the legs, the theoretical power necessary to move one of the actuators can be obtained from extracting the same power from the other actuator. This is logical since during the stance phase there is no increase in the robot elevation or speed, and the leg mass and friction have been disregarded.
- Even if it is physically possible to enlarge the step, it is advisable, from a energetic point of view, not to extend the legs excessively, especially at the end of the stroke. This way, the power can be kept within moderate values. In the case of step $S = 0.58m$, lengths of $L_{thigh} = L_{shank} = 0.65m$ should be used.

Most of actuators cannot transform the kinetic energy into electrical one, and therefore, the negative power cannot be recovered. Just on the contrary, the drive must supply the same power with opposite sign to extract the kinetic energy. Therefore, the energy that must be supplied during the support phase is

$$W_{support}^{coupled} = \int_{support} (|P_{hip}| + |P_{knee}|) dt \gg 0 \quad (12)$$

or, in case of having a braking resistor,

$$W_{support}^{coupled} = \int_{support} (\max(P_{hip}, 0) + \max(P_{knee}, 0)) dt \gg 0. \quad (13)$$

Since there is hardly any variation of the mechanical energy during the support phase, this energy must be considered as losses.

An extended analysis for the swing phase depends on the specific swing profile. In this phase, the inertia and leg weight are the terms under consideration, and their evolution will be shown when simulating the GC-VC leg during a complete step.

2.2 Losses in reptile-like (GC-LC) legs

There is an important amount of robot designs based on laterally connected legs, like Hirose's Titan series, SILO4, RoboClimber, Wheeleg, DanteII or Mecant. They entail a natural structure for small animals, but not in modern medium-to-high weight animals. In nature, this configuration is constrained to hexapod or octopod animals, or to reptiles that require an articulated body to extend the stride length during the crawl movement. Guardabrazo and Gonzalez de Santos (2004) state that generally, the mammal-type leg configuration is more suitable to support heavy loads than the reptile one.

Figure 4 represents the performance of the first type of LC legs, the one of subfigure 1(b) during the support

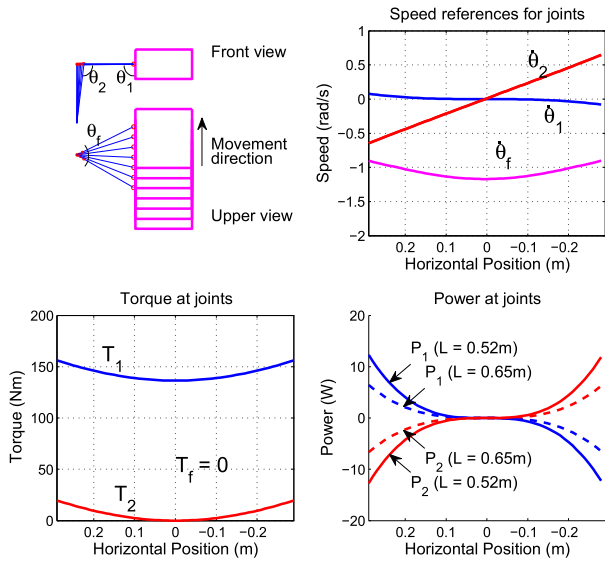


Fig. 4 Performance of a coupled reptile-like leg during the support phase: leg and body evolution, speed references, torques and power for the joints at the hip and knee.

phase. As expected, the actuator that must withstand the weight is moving, thus occasioning a non-null power that is considered as losses. Speeds, torques and power of the joints are also plotted for a leg with the same data as listed in table 1, except for the robot height that is the same as $L_2 \sim L_{shank}$. An additional pair of power plots has been included for lengths $L_1 = L_2 = 0.65m$, showing that the longer the legs the lower the power losses **StiffSpeed**, because a lower degree of coupling exists. In the limit, when $L_1 \rightarrow \infty$, a purely Cartesian performance is obtained. However, enlarging the legs also increase the robot width, as well as other effects in speed and autonomy that will be dealt with in section 2.4.

2.3 The proposed scheme compared to previous configurations

LegGuide The proposed decoupled leg (see figure 5) is the result of a complex synthesis procedure developed in Gonzalez-Rodriguez et al (2011), and it has been designed such that a constant (and frontal) velocity of the leg is achieved by only imposing a constant speed to the *frontal actuator* (Act_F). **LegGuide** The leg workspace has been obtained by means of the mechanism analysis program, MSC.ADAMS. Its large extent is shown in figure 6, where the origin of the coordinates is at the hip H . The thick lines show the relative trajectories of the foot when only one joint is acting. The foot trajec-

tory during the walking cycle must be included within the outer lines that represent the workspace limits.

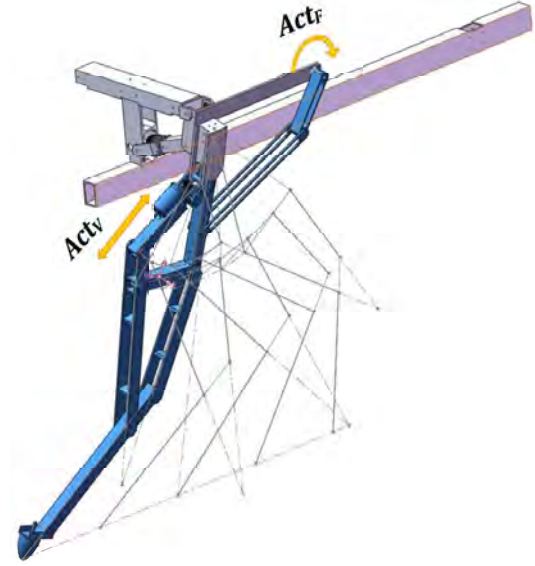


Fig. 5 Proposed leg (GD-VC).

The design conditions impose that:

1. During the support phase the *vertical actuator* (Act_V) does not move and hence $P_v \simeq 0$.
2. A constant reference is uniquely given to the *frontal actuator* that gives rise to a constant foot speed relative to the hip ($v_{foot}^{relat} = -v_{robot}$), thus ensuring that the foot does not slip.

To validate the assumptions for a quasi-Cartesian performance, a series of simulations have been performed in which the leg has moved in the frontal direction, while maintaining the vertical actuator in the constant position that it has during the support phase. The results are:

- The vertical coordinate of the foot end keeps an almost constant value, with a deviation of only $\pm 1.1mm$ with respect to the horizontality along the support phase, with a difference between maximum and minimum of $1.5mm$.
- The deviation of the frontal velocity is around $\pm 2\%$ in the central $500mm$.

This quasi-Cartesian performance entails an important advantage over most of the legs found in previous papers, whose two main drawbacks lie on their low energy efficiency (Alexander, 2005) and control complexity. An important reason for these drawbacks is that during the frontal movement of previous legs, the application of the inverse kinematics translates its required

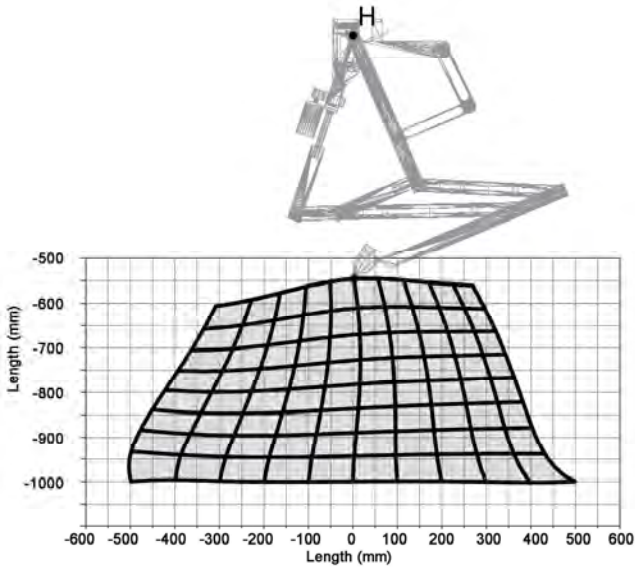


Fig. 6 LegGuide The leg workspace. Thick lines show the relative trajectories of the foot when only one joint is acting. The leg is drawn in its most retracted position.

constant velocity into continuously variable speed motions for all the involved actuators (two in a planar motion).

These drawbacks are present not only in GC legs, but also in GD-LC legs because they do not behave as Cartesian manipulators. As a consequence, a constant-speed straight horizontal trajectory requires that two actuators must be moved in a coordinated way following variable speed references, which in turn imply that they had to continuously waste energy both in accelerations and braking without possibility of recovering the kinetic energy. With this new leg design in which the inverse kinematics is reduced to a simple scaling, a constant actuator speed guarantees a constant foot speed in the forward direction, and hence the energy is only spent in overcoming the friction forces.

From the point of view of the control unit actuation, in the proposed leg the speed regulation task is easily accomplished by sending a reduced number of commands to the commercial electronic cards that drive the motors. On the contrary, existing robots require the calculation of the inverse kinematics and a trajectory-tracking control that demands a high frequency response, precision, and complex mathematical calculations (Collins et al, 2005). These requirements entail a complexity that often makes it necessary to use a control unit for each joint or a group of them.

With regard to the *frontal actuator*, simulations have been performed along the support phase of the proposed leg to show that its center of mass does not significantly vary its kinetic energy or height. Therefore,

and analogously to coupled configuration, the gravitational and inertia terms can be neglected.

This result and the fact that this actuator does not bear the load means that, during the support phase, this actuator only supplies the force/torque that compensate the friction. Disregarding it, $P_h \simeq 0$, and

$$W_{support}^{proposed} = \int_{support} (|P_v| + |P_h|) dt \simeq 0 \quad (14)$$

The four-bar mechanism that can be seen in the upper part of the leg, together with a linear actuator responsible for the vertical movement, increases the rigidity of the leg and reduces the hip height drop Δh , thus reducing the loss of gravitational energy.

In this scheme, a low-ratio gearbox has been used for the horizontal movements, thus allowing a higher speed and lower friction. On the other hand, a ball-screw mechanism has been selected for vertical movements, where the speed is not a limiting value, although a higher force to bear the robot weight is required. In addition, a vertical force in the foot is converted into a small force on the corresponding actuator, which allows the actuator to withstand the weight with reduced (Joule) energy consumption. This adequate actuator selection allows the designer to use lower capacity motors while maintaining the required specifications.

From the mechanical point of view, four additional advantages have been obtained:

1. As in the case of industrial robot arms, the actuators are located in the hip (or close to it), far from the end of the leg, thus reducing the inertia and wiring. In the case of LC legs, mainly the types of subfigures 1(b) and c), the inertia is higher for the same leg length, since more than half of the mass leg is located at the same distance as the foot. This issue entails an additional source of speed limitation for LC legs.
2. The reliability of the joints has been increased, because the actuators are out of the lines along which the impact and support reactions are directed.
3. The vertical and lateral actuators are prismatic ones with a screw mechanism, which constitutes an alternative to the harmonic drives for reduced backlash actuators. As a consequence, the cost of the presented robot is drastically reduced. It also allows including hydraulic actuator in the case of scaling to increased sizes.
4. The connection between leg and hip is vertical (like mammals), instead of being lateral (like insects or reptiles); therefore, the capacity load is improved and the width is reduced. The height also increases, and therefore the specific resistance improves (Jin et al, 2013).

A rubber support has been added to the leg to avoid slipping and alleviate the impact at the beginning of the contact. When the support is bearing the robot weight, the loads become higher and the rubber stiffness increases (Schaefer, 2009), which reduces its compression.

Additional mechanical details regarding the advantages, synthesis process, and simulation results of the leg structure can be found in Gonzalez-Rodriguez et al (2011, 2009).

A configuration with a similar performance is the vertically-connected two dimensional pantograph leg (subfigure 1e)). Derived from its pantographic structure, it presents locking problems when it is subject to lateral forces. The 2700kg ASV from Waldron and McGhee (1986) falls into this scheme. Many of the aforementioned advantages of the proposed leg are present in this configuration, except the one related to the joint reliability.

2.4 Effects of increasing the leg length

Drop Figures 3 and 4 showed that long legs can reduce the power exerted by the actuators. However, an excessive increase of the leg lengths may negatively affect the robot speed and efficiency.

With regard to the speed, the robot speed is limited by: the speed of the frontal movement; and the required time to separate the foot from the ground. This time is responsible for the overlapping or double stance in the support phase, and it increases the duty factor β , defined as the ratio of the durations of the support phase and swing phase. Therefore, in crawl gaits $\beta > 0.75$ for quadruped robots and $\beta > 0.5$ in hybrid or hexapod robots. The lift-off time depends on several factors such as the speed of the involved actuators, or their controller gains, but also the leg rigidity and the leg pose.

From a practical point of view, another limitation arise from the availability of gearboxes with the required output torque.

2.4.1 Effect of the rigidity in GC-LC legs

At the end of the stance phase, the vertical actuator begins to raise the foot but it does not leave the ground until certain additional rotation angle, $\Delta\theta_1^{lift-off}$, is reached (see subfigure 7(a). This angle is due to the link bending and the joint clearances, as well as the support compression, e_{compr} . Both the angle $\Delta\theta_1^{lift-off}$ and the inverse of the maximum rotational speed $\dot{\theta}_{1max}$, increase with the leg length L_1 . This way, long legs require more time to separate the foot from the ground before swinging the leg forward.

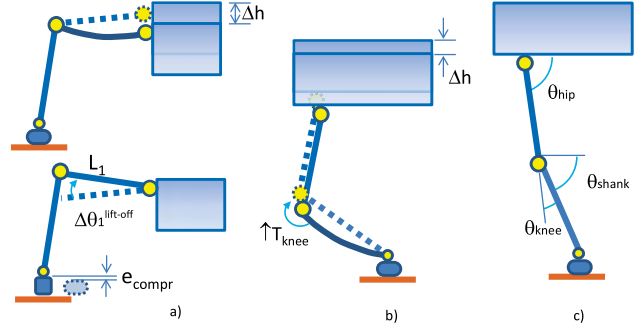


Fig. 7 Delay in the foot lift-off due to: leg bending and joint clearances in a) robots with GCLC legs and b) in robots with GCVC legs; c) difficulties achieving high vertical speeds.

The lack of rigidity, which is more prominent in long legs, also gives rise to additional mechanical losses during the support phase: in order to compensate the height drop Δh of the robot, the vertical actuator must be increased by a certain amount $\Delta\theta_1^{extens}$, thus causing an additional loss of energy.

$$E_{loss}^{grav} = M_s g \Delta h = M_s g L_1 \Delta\theta_1^{extens} \quad (15)$$

where M_s is the share of the robot weight that the leg must bear (the robot weight divided into the number of supports).

2.4.2 Effect of the rigidity and pose in GC-VC legs

As illustrated by Figure 3, longer legs can be used to reduce the high values for P_1 and P_2 . In this situation, the designer can either maintain the robot height, or increase it.

- If the robot height is kept constant (see figure 7(b), a longer leg gives rise to higher T_{knee} . In the same way as exposed in previous subsection, increased height drop, lift-off time, and energy loss are expected.
- As an alternative, if the robot height is increased in the same ratio as the leg length (figure 7(c), then thigh and shank are close to the vertical at the end of the stance. In this situation,

$$\begin{aligned} \sin(\theta_{hip}) &\gg 0 \\ \sin(\theta_{shank}) &\gg 0 \\ \sin(\theta_{knee}) &\simeq 0, \end{aligned} \quad (16)$$

and from expressions (4,5) high angular speeds are required to obtain a sufficient vertical velocity to speed up the foot ascent.

Therefore, unless another joint is used as an ankle, long legs also enlarge the lift-off transient.

A similar reasoning is also applicable to the proposed GD-VC leg. As a consequence, only the central 500mm of the theoretical 1m workspace is used.

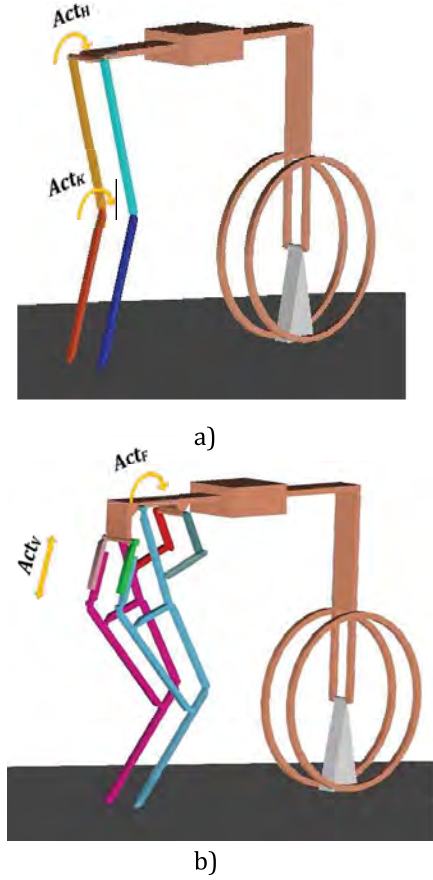


Fig. 8 ADAMS models of the hybrid robots used to compare the performance of coupled and decoupled legs

3 Energetic comparison as part of a hybrid robot

This section presents a set of numerical simulations performed with MSC.ADAMS, where both the mammal-like (GC-VC) leg and the proposed (GD-VC) leg are included in corresponding hybrid robots (see figure 8). The simulation results are presented along two complete steps of the hybrid robot. As in the majority of the papers, the lateral movement will not be taken into account.

The robot consists of two articulated legs and two passive wheels, and its speed is 0.2 m/s .

The rubber support has been modelled in ADAMS as a [contact characterized by a non-linear strain-stress curve](#). Their parameters have been selected after measuring, in the real prototype, the height drop of the hip when the robot is supported on only one leg and the wheels. The resulting value was $e_{compr} = 10 \text{ mm}$ and it is due to the rubber compression, the link flexion, and the joint clearances.

3.1 Control structure

The walking cycle can be viewed as a discrete event process, with few movement commands and transitions that can be easily identified, as in (Raibert et al, 1986). Its representation by means of Petri nets or Grafcets makes it very simple to control several processes (joints or legs in our case) simultaneously and its implementation can be easily included in the simulation or actuation cycle. If necessary, its implementation can be exported to a different platform (PC, programmable logic controller or PLC, microcontroller) or programming language (Narahari, 1999). The control structure consists of the following levels:

- At the high level of robot control, a discrete event control algorithm defines and imposes the actuator trajectories to complete a walking cycle, in a similar way as exposed by Maufroy et al (2010). Due to its smoother performance, sinusoidal trajectories (see Appendix) are preferred for both position profiles or velocity profiles.
- At the low level of each actuator, a speed PI controller establishes the velocity according to its calculated reference. An integral compensation of the position has been added to the PI controller that is equivalent to including a position PI controller. The resulting controller gains have been selected by balancing higher efficiency (lower gains) and sufficient speed and accuracy (higher gains).

3.2 Speed considerations about hybrid robots

Hybrid systems exploit the terrain adaptability of legs on rough terrain and simpler control as well as high speeds associated with wheels. In order to keep the static stability, multilegged systems are forced to be supported on at least three points. The polygon formed with these support points must contain the robot weight projection. In a legged robot, the stride is the distance the centre of gravity translates during one complete locomotion cycle Kar et al (2003), and it can also be seen as the absolute distance travelled by the swinging leg during the transfer phase.

The white bars of the chronographs of figure 9 represent the time intervals for the transfer phase, both for a quadruped robot (upper subfigure) and a hybrid one (lower subfigure). The robot stride is twice the leg stroke, which is the distance (relative to the hip) travelled by the leg during the support phase. This is also the distance travelled by the robot during the support phase. This phase corresponds to the black bars. The leg stroke, and proportionally the robot stride, are determined by the operative workspace of the leg. The

maximum horizontal speed is restricted by the maximum frontal speed and accelerations during the swing phase, which in turn are determined by the leg inertia and the frontal actuator capacity (including power electronic card, motor and gearbox). Therefore, since during the swing phase, the distance travelled, the maximum speed and accelerations/decelerations are predetermined for a specific leg, so it is the minimum time corresponding to the white bars.

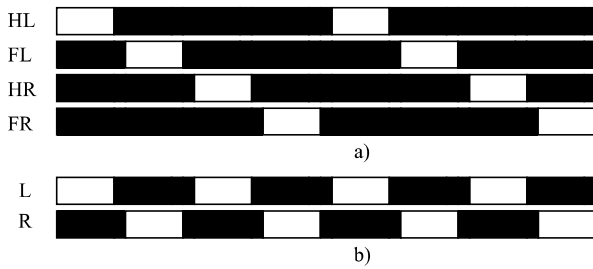


Fig. 9 Chronograph describing a) a possible walking pattern for quadruped robots and b) a pattern for hybrid robots. White bars refer to the transfer phase and solid black bar to the support phase. L = left, R = right, F = front, H = hind.

The upper chronograph represents two complete walking cycles for a four-legged robot. It shows that the duration of the support phase (black bar) is more than three times the duration of the swing phase (white bar). It could be exactly three times if there is no overlapping support, i.e. the robot is always supported on three points. The overlapping can be reduced if the leg is sufficiently stiff. Since three legs must be in the support phase (in the ground) as only one is advancing, it implies that during the support phase of one of the legs, all of the other legs must successively complete their swing phases. This increases the time invested in completing the locomotion cycle to four times the transfer phase, plus the corresponding overlapping.

If wheels are added to a two-leg configuration, they continuously provide two support points. Therefore, only one leg must be in the support phase as the other one is in the swing phase. If overlapping is not considered both phases require the same time. A situation with overlapping is represented in the lower chronograph. It shows that in the same timespan that the quadruped robot completes two walking cycles (equivalent to two strides), the hybrid robot completes four walking cycles (four strides). Assuming the same maximum horizontal speed as for the four legged robot, we can conclude that hybrid robots are, in theory, twice as fast as four-legged ones. StiffSpeed The previous explanation is resumed

in the expression

$$v_{robot} \propto v_{transfer} \left(\frac{1}{\beta} - 1 \right) \quad (17)$$

where $v_{transfer}$ is the maximum speed of the swinging leg, and β is the duty factor. Previous expression can be replaced into an equality if acceleration/deceleration times are neglected. Hybrid robot, as well as hexapod robots, can walk with $\beta \gtrsim 0.5$ (or equal if overlapping is neglected), while $\beta \gtrsim 0.75$ in the crawl gait of quadruped robots. This expression highlights the importance of reducing the double stance, and hence the lift-off transient, in order to achieve increased robot speeds.

Another important advantage of hybrid robots comes from its good stability. Since there are two aligned supports it is straightforward to achieve static stability. If the leg supports move in a line that is parallel and close to the sagittal plane, the stability is assured. Separating the walking legs would impair the stability margin.

3.3 Energetic comparison of GC-VC legs and GD-VC legs

Figure 10 represents the evolution of the energy consumed by the right leg actuators along two steps. As can be seen, most of the energy is consumed during the support phase, and it is quite higher in the GC-VC leg. The knee and hip actuators approximately consume the same energy, while in the proposed GD-VC leg, the consumption of the vertical actuator prevails. Simulations Figure 11 represents the torque/force of the actuators (upper plot) and their velocities (central plot). Lower plot represents the vertical actuator displacement and the height of the center of mass (CoM). Please note that the forces, torques and speeds, angles and positions are referred to actuators. In order to translate from actuators speeds to foot speeds, the following scales must be applied:

$$\begin{aligned} \text{for the vertical movement: } scale_v &= 10 \\ \text{for the frontal movement: } scale_f &= \frac{10\pi}{180} \text{ mm/deg.} \end{aligned} \quad (18)$$

In the case of the proposed leg, the consumption of the vertical actuator is theoretically null. However, it is not because there appears a height drop Δh that is due to several reasons:

- In the numerical simulations, the contact penetration. In the real robot, the lack of rigidity due to the link elasticity, the joint clearances and the rubber foot support.

- The hip height drop due to reduced gains in the vertical actuator controller (high gains give rise to lower efficiency).
- The fact that the proposed leg does not behave like a perfect Cartesian mechanism. As previously mentioned, an additional amount of up to $1.5mm$ must be added.

The situation is similar to that of walking in the sand, which requires more energy. [CoM] Therefore there is a consumed energy per leg and cycle that is equal to:

$$E_{loss} = Mg\Delta h_{CoM} = 84 \cdot 9.81 \cdot 0.012J = 9.89J \quad (19)$$

where Mg is the robot weight and Δh_{CoM} is the CoM height drop. As can be seen, the calculated value is similar to the energy consumed by one leg during the support phase as obtained from simulations (see sub-figure 10(b)). [HeightDrop] This height drop must be compensated by an additional extension of the leg Δl_{extens} , that is accomplished by the weight-bearing actuator. Its value is obtained as the sum of two addends:

- The hip height drop, which is twice the CoM height drop. Its maximum value is $\Delta h_{hip} = 2 \cdot 12mm = 24mm$.
- The contact penetration, e_{compr} , that models the compression of the rubber support. Its value is $10mm$ when only one front leg is supported.

Therefore,

$$\Delta l_{extens} = \Delta h_{hip} + e_{compr} = 34mm \quad (20)$$

In order to obtain this extension, the vertical actuator must be moved by an amount of $34mm/scale_v = 3.4mm$. Central plot of figure 11 shows the speed profile of the vertical actuator. The circled negative profile represents this height compensation, and the result of integrating this speed profile is depicted in the lower plot.

With regard to the rubber support, it has been modelled by means of a non-linear strain-stress curve with force exponent equal to $e = 2.2$. If no hysteresis is considered, the force is conservative, and the elastic energy during the support is obtained by integrating the force curve:

$$E_{elastic} = \frac{Mg}{2} \frac{e_{compr}}{e+1} \quad (21)$$

which only takes into account the robot weight borne by the supporting leg, $Mg/2$.

This energy is recovered during the foot lift-off. However, strain-stress curve does not perfectly obey Hooke's law, and a hysteresis phenomenon takes place. Hysteresis impedes stress force to be conservative, and therefore an energy loss will appear in every stress cycle.

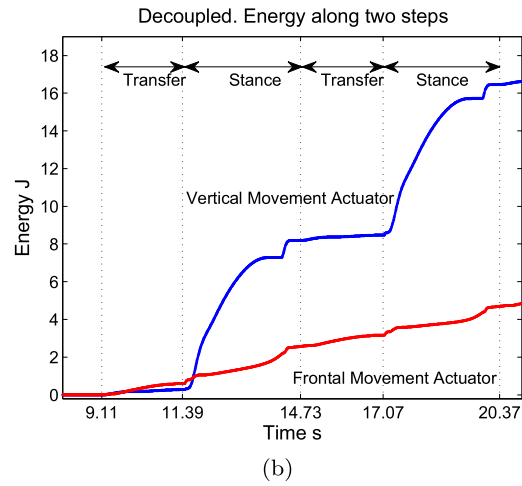
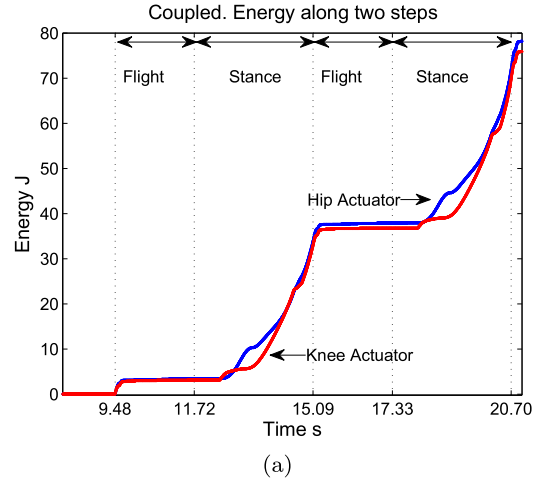


Fig. 10 Evolution of the energy of the right leg actuators along two steps in a) a traditional coupled GC-VC leg and b) the proposed decoupled GD-VC leg.

	Coupled	Decoupled
$E(J)$ with R_{brake}	$E = 78.7J$	$E = 19.6J$
$E(J)$ without R_{brake}	$E = 146.9J$	$E = 21.0J$
$\max(P_1)(W)$	$81.2W$	$94.9W$
$\max(P_2)(W)$	$135.38W$	$10.2W$

Table 2 Simulation results. Energy consumption of two legs along one cycle, and maximum capacity required ($=\max \text{torque/force} \cdot \max \text{speed}$). In the coupled scheme, 1 = *hip* and 2 = *knee*; in the decoupled scheme, 1 = *vertical actuator* and 2 = *frontal actuator*.

This energy loss has not been taken into account in the analysis.

Table 2 summarizes the result of the simulations performed over the two hybrid robots: the one with traditional GC-VC legs and the one with the proposed decoupled legs. The figures refer to the overall energy of both legs along one complete walking cycle.

It is worth mentioning that raising the robot speed would lead to proportionally raising the speed of both

actuators in the GC case, while only the frontal actuator speed must be raised in the proposed case. This fact leads to two important conclusions:

- There is a reduction in price and weight since a reduced-capacity frontal actuator can be selected.
- Increasing the speed leads to increasing the capacity of both actuators in the coupled case. Therefore, this speed is limited to the capacity of $150W - 250W$, which is the maximum range for most of the current intelligent servo-drives. In the case of decoupled legs, there is still enough margin to increase the robot speed, since it is mainly determined by the frontal actuator capacity.

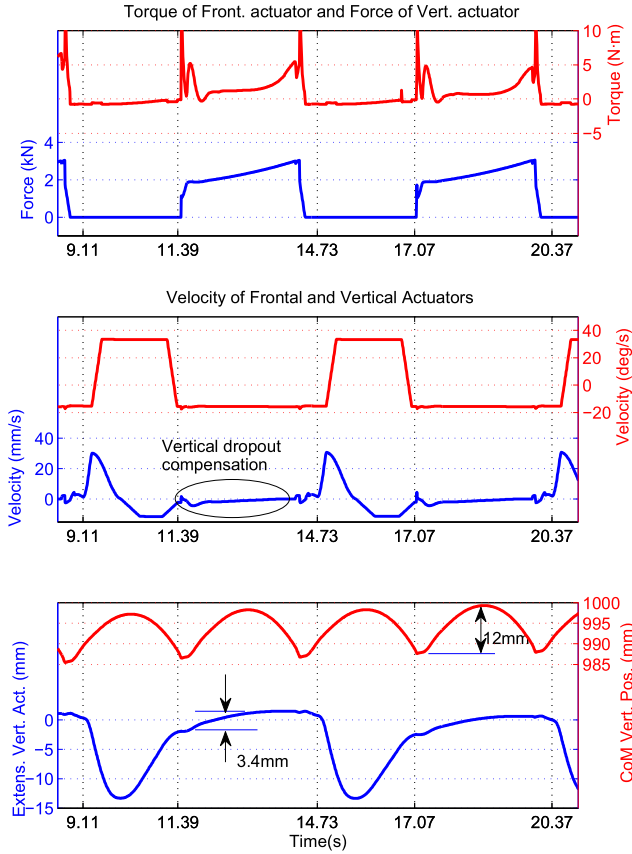


Fig. 11 Torque and force of actuators (upper plot); velocity of actuators (center plot); vertical extension and hip height (lower plot).

Assuming that the robot's weight is 84 kg and the stroke is 0.5 m, the mechanical cost of transport for an-

imal locomotion is

$$c_{mt} = \frac{\text{mech energy}}{\text{weight} \cdot \text{distance}} = \frac{21.0 \text{ J}}{84 \cdot 9.81 \cdot 0.5 \text{ Nm}} = 0.05 \quad (22)$$

similar to that in humans (0.05) or passive-dynamic walkers (0.055 for the Cornell biped and 0.08 for the Delft biped as expressed in Collins et al (2005)), but lower than the values for the hexapod (0.25) of Jin et al (2013), or the estimations made over the humanoid ASIMO (1.6) or NAO (2.4).¹ Gregorio et al (1997) gives additional values, although for the specific resistance.

3.4 Electrical losses and c_{et}

ELLosses Simulations have shown an improved performance in terms of the c_{mt} . However, in order to calculate the robot autonomy, the c_{et} is preferred, which involves the electrical energy consumption instead the mechanical energy. The difference between them is due to electrical losses, the motor friction and the reducer friction. Electrical losses consist of Joule losses in the dc motor armature, the losses of the electronic card, and the additional losses due to the fact that the PI output does not exactly follow the speed references. These two last components can be neglected if reasonable PI-gains are used.

Rated Power	$P_n = 150W$
Rated Voltage	$V_n = 24Vdc$
Rated Current	$I_n = 5.97A$
Armature Resistance	$R_a = 0.317\Omega$
Torque Constant	$K_{torque} = 0.0302 \text{ Nm/A}$
Speed Constant	$K_{speed} = 33.1962 \text{ rad/sV}$
No load current	$I_{nl} = 0.137A$

Table 3 Electrical parameters of a commercial DC motor.

In the case of a rotational joint coupled to the motor shaft through a gearbox

$$\Omega^{mot} = \Omega i^{gb} \quad T^{mot} = T / i^{gb} \quad (23)$$

$$T_{loss}^{red} = T^{mot} \frac{\eta^{gb}}{1 - \eta^{gb}}$$

where Ω and T are the angular speed and torque in the low-speed side, i is the reducer ratio, η its efficiency, and gb , red and mot stands for *gearbox*, *reducer* and *motor side*, respectively. For a prismatic joint that transform

¹ Please note that the calculated value is for a **simulated hybrid robot** and the remaining ones correspond to real biped or hexapod ones.

the rotational movement into linear one through a ball-screw mechanism

$$\begin{aligned}\Omega^{mot} &= V/i^{bs} \\ T^{mot} &= F/i^{bs} \\ T_{loss}^{red} &= T^{mot} \frac{\eta_{bs}}{1 - \eta_{bs}}\end{aligned}\quad (24)$$

where V and F are the speed and force in the low-speed side. The power losses can be calculated as

$$\begin{aligned}fem &= \Omega^{mot}/K_{speed} \\ I_{eff} &= T^{mot}/K_{torque} \\ T_{rot}^{mot} &= K_{torque} I_{nl} (fem/U_n)^2 \\ I_{rot} &= (T_{loss}^{red} + T_{loss}^{mot})/K_{torque} \\ I &= I_{eff} + I_{rot} \\ P_J &= I_{eff}^2 R_a \\ P_{rot} &= I_{rot} fem \\ P_{J_{rot}} &= (II_{rot} + I_{eff} I_{rot}) R_a\end{aligned}\quad (25)$$

where P_J are the Joule losses assuming no friction, P_{rot} is the rotational losses due to friction in the motor and the reducer (gearbox or ball-screw), and $P_{J_{rot}}$ are the additional Joule losses due to the existence of friction.

Table 3 represents the set of parameters of a commercial 150W permanent-magnet DC motor. A ratio of 750 will be used as a basis for the gearboxes in the GC-VC legs, and 74 for the rotational joint (frontal actuator) of the proposed legs. A basis of 35 000 rad/s : m/s is used for the ball-screw mechanism in the prismatic joint (vertical actuator) of the proposed legs. Constant efficiencies have been assumed for both reducers: $\eta_{gb} = 0.70$ for the gearbox and $\eta_{bs} = 0.50$ for the ball-screw. A set of simulations has been performed in which the reducer ratios have been multiplied by 0.5, 1 and 2. Table 4 shows the distribution of energy both for a robot with GC-VC legs and for a robot with the proposed leg, for both legs along a complete step. The analysis of the energy losses allows the designer to modify the speed trajectories and to select the reducer ratios.

The electrical cost of transport yields (ratios 35 000 and 74) yields

$$c_{et} = \frac{E_{elect}}{weight \cdot distance} = \frac{72.32 \text{ J}}{84 \cdot 9.81 \cdot 0.5 \text{ Nm}} = 0.18 \quad (26)$$

Different issues can be extracted from this analysis:

- As expected, higher reducer ratios increase the rotational losses. Lower ratios increase the Joule losses
- The terms $\sum P_J$, $\sum |P_{J_{rot}}|$ and $\sum |P_{rot}|$ cannot be considered as losses in all situations. Actually, summing these terms and the mechanical energy does

Robot with GC-VC legs						
Joint	Hip	Knee	Hip	Knee	Hip	Knee
ratio	375	375	750	750	1500	1500
$E_{mech}(W)$	146.89		146.89		146.89	
$\sum P_J(W)$	104.66	54.04	14.29	13.26	3.57	3.29
$\sum P_{J_{rot}} (W)$	49.26	53.55	12.13	13.29	3.20	3.61
$\sum P_{rot} (W)$	32.71	32.27	35.03	36.90	44.27	55.17
E_{elect}	296.18		191.58		184.32	

Robot with the proposed leg						
Joint	Vert	Front	Vert	Front	Vert	Front
ratio	17 500	37	35 000	74	70 000	148
$E_{mech}(W)$	20.97		20.97		20.97	
$\sum P_J(W)$	33.98	15.55	8.49	3.89	2.12	0.97
$\sum P_{J_{rot}} (W)$	58.61	16.15	14.74	4.06	3.80	1.03
$\sum P_{rot} (W)$	17.24	2.17	21.36	2.21	37.83	2.35
E_{elect}	158.82		72.32		66.28	

Table 4 Mechanical power and losses for a robot with GC-VC legs and with the proposed legs. Losses have been calculated for three different set of reducers per robot.

not yield the final electrical energy E_{elect} . This is due to the fact that, when the joint is being decelerated, R_a and the friction/windage torque do not increase the torque; on the contrary, they reduce the required braking one.

- The high values of torque required by the joints in the GC-VC leg during the support phase entail high gearbox ratios, which increase cost and weight/inertia.
- Other constraints must be taken into account to select the reducer ratios: an excessively high ratio gives rise to speeds in the motor side that will exceed the rated one; an excessively low ratio gives rise to currents in the motor armature that will produce overheating. ReducerSpeed As a consequence, in both GC-VC actuators and in the prismatic one, an increase in the speed of the the actuators would lead to unacceptable energy losses unless a higher capacity dc motor (and correspondingly a bigger reducer) is used. However, the frontal actuator can easily increase its speed without any modification, thus allowing higher speeds in the proposed leg. Actually, the speed of the real prototype is 4.5 times that of the simulations, as will be shown in next section.

3.5 Additional considerations

As stated by McGeer (1990b) flexing knees allows the robot to bring forward the leg that is in the air with reduced help of motors. This way, the simulation for the mammal-like (GC-VC) leg shows a better performance along the swing phase than the proposed leg. However, it does not balance the drastic efficiency improvement

of decoupled legs during the support phase.

During the support phase, the energy consumed by the frontal actuator is much smaller than by the vertical actuator. This is only possible if the robot do not accelerate or decelerate during the walking cycle. To that end, during the support phase the frontal actuator must guarantee that the foot frontal speed relative to the hip is

$$v_{foot}^{relat} = -v_{robot}. \quad (27)$$

The control algorithm must assure that the foot satisfies this condition not only during the support phase properly said, but also before coming into contact with the ground, and also until being sure that the foot has taken off from the ground. To this end, pressure sensors should be included in the foot of real robots to notify the unit control of the impact and the lift off.

Furthermore, in the case of mammal-like (GC-VC) legs, they may bend the knees in the opposite direction of travel (such as humans), or in the same sense, as ostriches. In the case of unsymmetrical decoupled legs, like the one proposed, it is also possible to select the leg orientation. Unless other circumstances arise, it is preferable to dispose the leg such that the stiffer pose (in terms of vertical forces) occurs when the leg is closer to the center of mass.

In addition, low values for the controller gains smooth over the actuator outputs, which also contributes to reduce Joule losses.

4 Experimental results – Controlling the real hybrid locomotion robot

The prototype of figure 12 has been built. As can be seen the prototype includes the proposed GD-VC leg, both in the front and hind side. It also includes lateral actuators to assure balance. The hind legs, as well as the lateral actuators, are not necessary during normal walking on flat terrains. [Weight] Its total weight is 84kg, that can be lowered by using aluminium instead of steel in the chassis and hip, and simplifying the lateral actuator design. Each leg weights 7kg, excluding the frontal and lateral actuators, that are attached to the hip. As a reference, the weight of similar size robot is: Titan VI, 190kg; Silo6, 44kg; Rimho, 65kg; hexapod of Jin et al (2013), 50kg, with 5.5kg/leg.

The joints are actuated by means of DC motors. The prototype includes an electronic drive per motor that are provided with the trajectories that the connected motors are forced to follow. The exact time at which



Fig. 12 The hybrid robot including the proposed decoupled leg.

every trajectory must be initiated is given by the accomplishment of certain conditions, which are the transitions of the Grafcet or Petri net.

The quasi-Cartesian performance of the proposed decoupled leg allows the definition of the leg trajectory by means of only one (or none) movement command per state. On the contrary, in the case of the support phase of a coupled leg, the references are variable in time, which forces the inclusion of low level functions to make the actuators follow the desired variable references. This fact complicates the robot control, and saturates the communication channels with continuous commands between the unit control and the electronic drives.

In the case of the presented prototype, the control simplicity (five commands per leg and cycle) and the absence of trajectory tracking entails a good option for fast-walking robots. The calculations necessary to coordinate the legs are straightforward and take less than 1 ms, while each of the five trajectory commands takes less than 5 ms. In this way, a swift leg operation can be obtained with reduced time dedication from the unit control, whose main task is to supervise whether the transitions are fulfilled.

Taking advantage of these characteristics, a robot speed of $v_{robot} = 0.9 \text{ m/s}$ has been achieved, which is 4.5 times higher than that obtained in the simulations. [OtherSpeed] The proposed leg and the hybrid structure allows the robot to be significantly faster than most of existing electric-driven robots in this size: hexapod from Jin et al (2013), 0.35 m/s; Turtle 0.05 m/s; Silo6, 0.05 m/s; QU 1120 from HTR, 0.12 m/s estimated.

The movement of the real prototype has been captured using an OptoTrack system. The obtained data has been reported in Gonzalez-Rodriguez et al (2011), showing a similar performance to that expected from

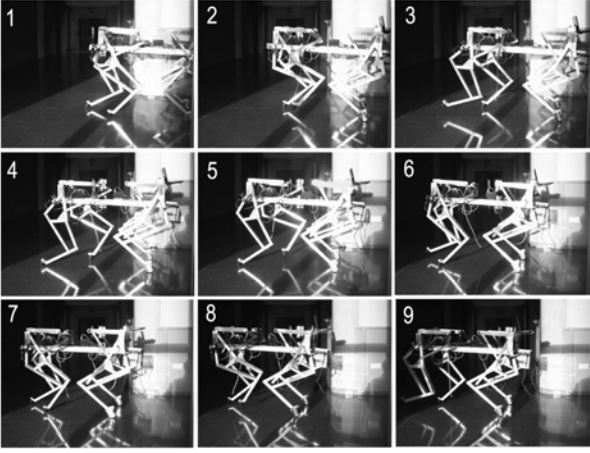


Fig. 13 Sequence of movements of the hybrid mobile robot.

the simulations. However, the differences appear mainly due to lack of rigidity of the links and clearance between the motor shafts and the aluminium links. They force the foot elevation to be increased from 100 mm (as used in the simulations) to 200 mm (in the real prototype). To reduce these differences, a second version of the robot was built with a stiffer structure, which reduces the hip height drop, and therefore the necessity for its compensation $\Delta l_{\text{extends}}$, of expression (19). This leads to a lower duration of the transfer phase, a higher robot speed, and lower energy losses.

Figure 13 shows a sequence of snapshots taken from the first version (not stiffened) of the hybrid robot. Videos of the walking process of the hybrid robot (called LEgged Bioinspired Robot with Efficient operation Limbs or LEBREL) can be found at www.lebrel-robot.com ($\rightarrow$ Multimedia), both for the original robot and the improved stiffness version.

The webpage also includes a sequence of the robot overcoming a 20 cm obstacle and climbing two steps up and down. These last operations require enabling the hind-leg joints as well as the lateral actuators to maintain balance.

The experiments did not show a significant deviation of the robot from straight trajectory. However, in a general case, sensors or a computer vision system should be included, which can estimate the real robot positions starting from its relative position with regard to fiducial hints. This feedback allows the control unit to adjust the trajectory in an adaptive way.

5 Conclusions

This paper compares the performance of different type of legs in terms of energy efficiency, and hence autonomy, as well as speed.

Firstly, in the second section, the comparison is focused on showing the drawbacks of GC legs during the support phase, demonstrating that inertia and leg weight terms can be disregarded in the power analysis.

Later, in the third section, two hybrid robots have been modelled and simulated: one including mammal-like (GC-VC) legs; and the other including the proposed (GD-VC) leg. Simulations show a reasonably more efficient performance of the hybrid robot with the proposed decoupled leg. Furthermore, there is a limitation in the speed of hybrid robot consisting of GC-VC legs, that is not present in the proposed configuration.

A part of the advantages of the proposed leg design are inherited from its GD configuration: a more adequate selection of actuators leads to a smaller motor with reduced consumption; the supplied power during the support mode is also reduced because the vertical actuator is the only one withstanding the gravitational forces, and its speed is zero during this mode; and the actuators are located far from the foot, thus reducing inertia, wiring, and vibrations.

Other advantage is that, in the same way that Waldron's ASV, Cartesian coordinate motion is obtained, thus reducing the inverse kinematics to a simple scaling.

A study has been included that highlights the influence of the leg length on the lift-off time, and hence, in the robot speed, as well as an analysis of the electrical losses in the motor drives.

The proposed decoupled leg has been included in a real prototype to operate the motors according to the calculated trajectories. Two main versions of the robot have been built. The stiffer one leads to lower differences between the simulated model and the real prototype, and higher speeds and efficiency can be obtained.

A Sinusoidal profiles

During the transfer phase (also called swing or flight phase), the movement of the frontal and vertical actuators have been simulated according to the following sinusoidal profile

$$v(t) = v_o + (v_f - v_o) \sin^2\left(\frac{t\pi}{2t_a}\right) \quad (28)$$

In this expression v_o is the initial velocity, v_f is the final velocity, t is the time from the beginning of the acceleration and t_a is the time required for $v(t)$ to reach the final velocity. The expression is also valid for the deceleration. In this case t_a can be changed into t_d .

The travelled distance during a sinusoidal accelerated profile is

$$e(t) = \frac{v_f + v_o}{2}t - \frac{v_f - v_o}{2} \left[\frac{t_a}{\pi} \sin\left(\frac{t}{t_a}\right) \right]. \quad (29)$$

For $t = t_a$, the distance travelled is the same as that in the case of the linearly accelerated profile. However, the maxi-

mum acceleration is

$$\max \left[\frac{v_f - v_o}{2t_a} \pi \sin \left(\pi \frac{t}{t_a} \right) \right] = \frac{v_f - v_o}{2t_a} \pi \quad (30)$$

Therefore, the same behavior for the linear and sinusoidal profiles can be assumed with the only concern being that the maximum acceleration in the latter profile must reach a higher value ($\pi/2$ times) than that in the linear profile. In this way, a sinusoidal profile does not add significant operations to the control loop, but gives way to smoother paths, thus improving the robot performance. Furthermore, the maximum acceleration that can be obtained (limited in most of the cases by vibrations) is higher in the case of sinusoidal profiles than in the case of linear profiles, which means that the minimum acceleration times are approximately the same in both profile types.

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